Combining Monte-Carlo Simulations and Options to manage Risk of Real Estate Portfolios

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Abstract

This paper aims to show that through the simultaneous use of Monte-Carlo Simulations and options theory, the accuracy of real estate portfolio valuations can be improved. Our method considers the options embedded in lease contracts, especially as conceded to tenant in continental Europe. We combine Monte-Carlo simulations for both the market prices and rental values with an optional model that takes into account a rational tenant’s behavior. We analyze to what extent the options exercise by the tenant significantly impact the owner’s income. Our main findings are that simulated cash-flows taking into account such options are more reliable that those usually computed by DCF traditional methods. Moreover this approach provides interesting measurements such as the cash-flows distribution, the probability of leaving for the tenants and the derived optimal holding period for the owner. The model also provides a risk measurement by computing the Value-at-Risk and a risk-adjusted performance measurement by computing a forward Sharpe Ratio of the considered portfolio.

After a brief review of literature on simulation methods used for real estate valuation, the paper describes the suggested simulation model, its main assumptions, and the incorporation of tenant’s decisions as break-options influencing the cash-flows. Finally, through an empirical example, we analyze the sensitivity of the model to various parameters and we test its robustness.

KEYWORDS: Monte-Carlo Simulations; Real Estate Portfolio Valuation; Break-option; Lease Structure; options

I. Introduction

Real estate investment is all about property or portfolio valuation. Therefore the investors in real estate require a perfect understanding of the real estate market and a perfect knowledge of the methods used to value real estate asset. Valuation is itself all about risks, uncertainty and opportunities. Fundamentally the main risks faced by real estate investors are the operating costs, the vacancy rate, the lease contract and the liquidity and on the other side, the main opportunities are the operating costs, the terminal value and the rental growth (leases often provide mechanisms to increase total payments over the course of the lease). The need for appraisals in real estate arises from the heterogeneous nature of properties: no two properties are identical, and all properties differ from each other at least in their location - which is one of the most important determinants of their value. So there cannot exist a centralized Walracian price setting for the trading of property assets, as there exists for securities in capital market. The absence of a market-based pricing mechanism proves the necessity to use relevant and reliable method of valuation.

Real estate valuation is the task of appraising the prospective price of a property. Real estate can harbor almost all pathologies encountered in the practice of valuation while at the same time, is traditionally considered a more predictable asset type. Following Nassim Taleb who underlined commodities' specificities in its foreword to Geman (2004), we emphasize in this introduction some of the specifics of real estate. First exception: the location. While a security is an abstract item, a simple balance sheet entry, properties present location characteristics that make arbitrage arduous and comparison difficult. Secondly, the temporal dimension. A real estate asset is severely illiquid. The action of buying and selling is hardly unpredictable in real estate. They are heavily grounded to their physical nature. Buying and selling spans a matter of months and even years for real estate assets in compared with a security which can be traded twice in a second. Thirdly, the investment size. Real Estate assets are large, non dividable assets. Fourthly, the obsolescence rate. A building doesn't keep its level of efficiency over the time. Fifth is the cash flow occurrence. Small cash flows occur during the holding period and large flows occur at the time of sale. Bearing all these hurdles in mind, the difficulties of real estate valuation become obvious.

The major traditional valuation methods well accepted by practitioners and academics are, among others, the cost of construction, the comparables, the yield capitalization, the discounted cash-flow or the asset present value. These traditional valuation methods suffer from numerous limitations and critics. But, in particular, they all suffer at least from the same inherent limitations: they do not appropriately take the risk into account and they are too sensitive to some parameters (infinite growth rate for instance). These limitations are discussed in Fama and French (1989) and Ferson and Campbell (1991) in addition of Myers (1974) who promotes the asset present value approach. Furthermore, the traditional valuation methods do not meet certain basic requirements such as error measures predictions, production of Value at Risk, the distribution of possibilities, the standard error calculation or the confidence interval. It is now sometimes essential to value risk of a real estate asset or real estate portfolio. These limitations of the traditional methods are really problematic and we assume in this paper to override all these issues by suggesting a new valuation method which, using Monte-Carlo simulations and options, incorporate uncertainty in the valuation process. Monte Carlo has long been applied to incorporate risk over the simulation of numerous scenarios, but adding an optional part allows us to also take into account the risk bared by the lease contract. Moreover methodology using simulations techniques enables one to derive a broad spectrum of applications and measurements calculations (such as Value at Risk).

The first introduction of simulation techniques for real estate assessment is found in Pyhrr (1973) who analyzes the risk of a real estate investment using simulation methods. Numerical methods are used, among others, by Wofford (1978), Mollart (1988), Li (2000), Dupuy (2003), Kelliher & Mahoney (2000), French & Gabrielly (2005), Baroni, Barthelemy & Mokrane (2005) and Hoesli, Jani and Bender (2006). The idea initiated by Pyhrr (1973) consists of making risk assessment more explicit and to make a better use of modern financial theory. Especially, in 1973, Pyhrr underlined how it is possible to use numerical methods to develop models that help the investment decision maker to take three dimensions into account: degree of uncertainty, time dependence and complexity. Its model forms the basis of all the modern models that use Monte-Carlo simulation.

Our paper takes principally a leaf out of three papers: that of Hoesli, Jani and Bender (2006), that of Baroni, Barthelemy and Mokrane (2005) and that of Dupuy (2003). These three papers are also – in our knowledge – the first feasible implementation in practice.

Hoesli, Jani and Bender in 2006 seek to add uncertainty in the valuation process and to solve the constant weighted average cost of capital ("WACC") hypothesis by using Monte-Carlo simulations. Precisely they simulate the risk free rate with the Cox, Ingersoll, Ross (1989) dynamic model and incorporate in this way variations in the risk free interest rate. Moreover the models proposed by the authors suggest adding some building specific characteristics in the risk premium. This is achieved through a rating of the states of the markets and an assessment of property specific hedonic characteristics which are translated into a building-specific risk premium. The proposed approach succeeds to override some of the worst limitations of the discounted cash flow model and besides others to incorporate risk in the cash flows and to use varying weighted average cost of capital. The best advance of this method is that it does not require prior knowledge of the asset's value (usually necessary to determine the weighted average cost of capital).

In 2005, Baroni, Barthélémy and Mokrane* proposed a new real estate portfolio valuation methodology that uses Monte-Carlo simulations. They proposed to simulate both the rental value and the price of the asset. Their model innovates by introducing uncertainty in the cash flows but also in the price of the asset. Furthermore, they model the vacancy rate using a Uniform law and thus also incorporate the possibility of

* Their article follows a first one published in 2001 using basically the same idea

vacancy changes in the portfolio. This article allows, among other things, to override one of the most profound issues in valuation: the terminal value which is simulated instead of being dependent on a hazardous infinite growth rate. Empirical tests using an author's built index proves how robust their methodology is in comparison to the traditional discounted cash flow approach. Likewise this approach allows the measurement of the risk given the outcomes distribution. Specifically, the value at risk can easily be computed. This paper also opens the way to numerous applications in terms of portfolio management, another publication of Baroni, Barthélémy and Mokrane (2005 b) derives the optimal holding period of a portfolio which is a classical difficult issue in Finance.
The first combination of both Monte-Carlo simulations and options for real estate valuation is introduced by Dupuy in 2003. The author considers the risk borne by the real estate owner focusing on the option granted to the tenant in the lease contract. In this way, Dupuy concentrates on market rental values ("MRV") and uses Monte-Carlo simulations for the market rental values combined to options to demonstrate how a traditional lease structure transforms normally distributed market rental value distribution into a reduced set of income paths. More precisely, the author derives the tenant's behavior by comparing tenant's cash flows and market rental value's cash flows expectations for each simulated scenario. In addition Dupuy derives numerous applications and measurements as of the average presence length in premises or the probabilities of a tenant to vacate.

Following Dupuy (2003) and Baroni, Barthélémy and Mokrane (2005), we undertake in this paper to enhance existing real estate valuation methodologies by introducing uncertainty and risk in the valuation process. This is reached by combining Monte-Carlo simulations with option's theory. The model we propose innovates by taking into account the risk underlying in the lease structure according to the level of the market rental values, the agency costs and the current rent paid. Precisely, we use Monte-Carlo simulations for the price and the market rental value and options for the break-options possibility granted to the tenant in order to confer a value to each lease given the simulated state of the market (for the market rental value and for the price) and given the lease structure. Our model innovates by taking into account the risk inherent in the lease structure at a portfolio level. Thus, the model we developed incorporates uncertainty in the level of cash-flows according to the state of the market rental values but also incorporates uncertainty in the price component. Our method claims to be more accurate and more reliable than traditional methods.

The article proceeds as follows. Firstly, we have briefly debated real estate valuation methods, their limitations and presented the existing literature on simulations methods used for real estate valuation. The following section develops the model and illustrates it is an innovative approach. In particular, we focus on the risk inherent in the lease contract that we model with options. The third section provides applications and illustrations of our model based on an academic case where we conduct a sensitivity analysis to measure the robustness of our model.

Throughout the paper, we assume the investors are rational and we use present value to decide about favorable investments.

II. The model

Our valuation model is designed to value any assets or portfolio, rented or not and with or without a lease structure. Our model makes use of Monte-Carlo to simulate numerous scenarios for the price of an asset and its market rental value. Then for each scenario, the simulated Market Rental Value for each lease and at the relevant dates is compared with the rent and the best tenant's rational decision (leaving, negotiation or staying) is taken and incorporated in the cash-flows. After that, all the results are aggregated to confer a value to the portfolio using a classical discounted cash flow model.

II.1. Simulation of the Price, the market rental values and the rents

Following Baroni, Barthelemy and Mokrane in 2005, our analysis leads us to simulate two components: the price of the asset and the market rental value together linked by a correlation factor. It is essential to differentiate the rent from the market rental value. We differentiate and define them as: the rent (R_t) is the price paid by a tenant at time t to a landlord whereas the market rental value at time t (MRV_t) is the value that the place or the space is worth at time t.
The rent is traditionally indexed (on CPI, specific index or other…) depending on the lease contract. The rents do not need to be simulated, instead, the indexes could be forecasted or estimated and the rents can easily be computed as soon as a relevant estimation of the indexes can be found.

The two simulated processes – the Market Rental Value ("MRV") and the Price – are assumed to follow a normal law. We recognise that this hypothesis is a strong one but it is generally accepted by practitioners. Thus the processes can be completely defined in terms of their trends and volatilities. Therefore the market rental value and the price are supposed to be governed by a geometric Brownian motion.
The Price and the MRV are modelled by a diffusion process:

$$\frac{dX_t}{X_t} = \mu_t^X dt + \sigma_X dW_t^X$$

Cash flows and Prices are simulated over long period and we decide to use time-varying trends. This is why the two trends vary over time and are driven by a piecewise function:

$$\mu_i^P(t) = \mu_i \times 1_{[t;t+1]} = \begin{cases} \mu_i, if\ i \in [t,t+1] \\ 0, if\ i \notin [t,t+1] \\ i \in \mathrm{N} \end{cases}$$

$$\mu_j^{MRV}(t) = \mu_i \times 1_{[t;t+1]} = \begin{cases} \mu_j, if\ \ j \in [t,t+1] \\ 0, if\ \ j \notin [t,t+1] \\ j \in \mathrm{N} \end{cases}$$

The volatility σ_P and σ_{MRV} are supposed to be constant over time.

Specifically the Price component is driven by:

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_P dW_t^P$$
$$dP_t = P_t \left[\mu_t^P dt + \sigma_P dW_t^P \right]$$

Then, the simulation period [0,T] is divided in sub-periods dt =[t, t+1] and we apply Ito's lemma to ln(Pt):

$$d(\ln P_t) = \frac{dP_t}{P} - \frac{1}{2}\left(\frac{dP_t}{P_t}\right)^2 dt$$
$$d(\ln P_t) = \frac{dP_t}{P} - \frac{1}{2}\left\|\sigma_p\right\|^2 dt$$
$$d(\ln P_t) = \left(\mu_t^P - \frac{1}{2}\left\|\sigma_p\right\|^2\right)dt + \sigma_P dW_t^P$$
$$P_t = P_0 \exp\left\{\int_0^t \left(\mu_t^P - \frac{1}{2}\left\|\sigma_p\right\|^2\right)du + \int_0^t \sigma_P dW(u)_t^P\right\}$$

Which finally gives*:

$$P_t = P_0 \exp\left\{\left(\mu_t^P - \frac{\sigma_P}{2}\right)t + \sigma_P W_t\right\}$$

$$P_t = P_{t-1} \exp\left\{\left(\mu_t^P - \frac{\sigma_P}{2}\right)\Delta t + \sigma_P \sqrt{\Delta t} \times U^P\right\}$$

In the same way, the market rental value simulation is driven by the following process:

$$\frac{dMRV_t}{MRV_t} = \mu_i^{MRV}(t)dt + \sigma_{MRV} dW_t^{MRV}$$

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* As shown by the equations our model requires the initial price of the portfolio. In this way our model presents the same issue that the traditional discounted cash flow as the price of the asset is itself used to determine the price of the asset. The well-known circular problem (see for instance Hoesli, Jani, Bender (2006)) of the inputted price remains the same.

which can be derived to:

$$MRV_t = MRV_{t-1}\exp\left\{\left(\mu_i^{MRV}(t) - \frac{\sigma_{MRV}}{2}\right)\Delta t + \sigma_{MRV}\sqrt{\Delta t}\times U^{MRV}\right\}$$

Given the dependence – the co-relation – between the price of a real estate asset and its market rental value, it is not possible to simulate the two processes independently. We thus need to consider their correlations. Even so more we consider in this paper the correlations between the market rental values and the price but also between the markets rentals values themselves. Indeed the price of a portfolio and all the market rental values that composed the cash flows are linked but also all the market rental values are themselves linked by nature. A bear market in the centre of London has a high probability to influence the markets of almost all others European countries. Therefore it is essential when considering a portfolio to take into account all the correlations that composed all the elements of the portfolio. The concept of correlation is a quantity that measures the degree of linear joint-variation between two variables. In our case, we seek to estimate how much the Price and the market rental value change together or contrarily and to estimate how much the market rental value of one sub-market impacts the market rental values of the other markets. In our case the correlation factor is supposed to be strong because prices of portfolios are greatly influenced by the level of possible cash flows and the markets have proved to behave globally. This factor – denoted $\rho_{X/Y}$ in the following – is supposed to be constant over time.

To obtain a couple (X_i, Y_i) which are linked by a correlation factor, the two processes must be simulated simultaneously using the following system:

$$\begin{cases} X_t = \mu_t^X + \sigma_X U_t^X \\ Y_t = \mu_t^Y(t) + \rho_{X/Y}\sigma_Y U_t^X + \sigma_Y\sqrt{1-\rho_{X/Y}{}^2}\,U_t^Y \end{cases}$$

Where U_X and U_Y are two standard normally independently distributed variables.
This system is derived from a Cholesky decomposition. We remind here the formula of this decomposition in the particular case of two processes only:

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \rho_{X/Y}\sigma_X\sigma_Y \\ \rho_{X/Y}\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix} = \begin{bmatrix} \sigma_X & 0 \\ \rho_{X/Y}\sigma_Y & \sqrt{1-\rho^2_{X/Y}}\,\sigma_Y \end{bmatrix} \times \begin{bmatrix} \sigma_X & \rho_{X/Y}\sigma_Y \\ 0 & \sqrt{1-\rho^2_{X/Y}}\,\sigma_Y \end{bmatrix}$$

This system makes us able to derive two correlated paths for a given period.
The Cholesky decomposition is commonly used in the Monte Carlo method for simulating systems with multiple correlated variables: the matrix of inter-variable correlations A is decomposed, to give the lower-triangular L. This is why it is necessary to use Cholesky decomposition to decompose our symmetric,

positive-definite matrix of correlation factor into a product of a lower triangular matrix and its conjugate transpose. Mathematically, if A has real entries and is symmetric and positive definite, then A can be decomposed as:

$$A = L \times L^T$$

where L is a lower triangular matrix with strictly positive diagonal entries, and L^T denotes the conjugate transpose of L. In our case, for n processes, the vector A has the following form:

$$A = \begin{bmatrix} \sigma_1^2 & \cdots & \rho_{1/i}\sigma_1\sigma_i & \cdots & \rho_{1/n}\sigma_1\sigma_n \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \rho_{1/i}\sigma_1\sigma_i & \cdots & \sigma_i^2 & \cdots & \rho_{i/n}\sigma_i\sigma_n \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ \rho_{1/n}\sigma_1\sigma_n & \cdots & \rho_{i/n}\sigma_i\sigma_n & \cdots & \sigma_n^2 \end{bmatrix}$$

Traditionally, the rent is indexed on a country-specific index such as the inflation rate, the consumer prices, the construction cost price, a specific index or a fixed growth rate. The rent is usually reviewed on an annual basis with, sometimes, an optional maximum or minimum barrier. In the case a barrier is present; the rent will be indexed either only if the index pass a certain level or for a maximum growth equal to the barrier if the index has shown an higher increase than the barrier over the period. More precisely, the index if it passes a certain level - the barrier - is either taken for update of the rent or rejected in favour of the barrier depending on the contract. Some specific cases exist as the upward-only review or the back to the market review. To anticipate the rental value, it is thus necessary to find a relevant forecast of the index. The rent paid at time t is therefore the composed indexation of the rents at time 0. Mathematically, the rents can thus be anticipated as taking the following form:

$$\begin{aligned} &R_T = R_t \times (1+r)^{T-t} \\ &R_t = R_{t-1} \times (1+r_t) \\ &R_t = R_{t-1} \times (1+\min(r_t,\mathrm{B})) \\ &R_t = R_{t-1} \times (1+r_t),\ r_t \geq B \end{aligned}$$

Therefore as soon as one can exhibit a relevant index forecast, it becomes possible to predict the future rents and thus the future incomes.

II.2. Taking the lease structure risks into account: the options

In Europe lease structures vary from one country to another and quite differently in UK. The lease structure is one of the key determinants of real estate risk. It is also an essential component of the cash-flow model. Fundamentally, a lease is a rental agreement between a landlord and a tenant. The lease provides detailed information about expected cash flows for years to come. Precisely the lease specifies, among other things:

the starting date, the initial rent, the end of lease, the indexation and the options granted to the tenant to leave the premise before the end of the lease. These options granted to the tenant are called the break-options. For the European landlord, the key risky triggers of the lease are the break-options. These two elements are very similar as they both are likely to cause vacancy, to incur lack in cash flows and thus are the main risks faced by investors. Basically at the time of a break-option or at the end of the lease a tenant has two possible choices: staying in the premise or leaving. But practice has shown different situations. In practice, landlords are likely to concede potential revision in the rent level in order to prevent void situation and, at the same time, tenants traditionally prefer to stay in the premise in order to save transaction* costs. So negotiations are common in real estate. To summarize threefold, at the time of a break-option or at the end of the lease a tenant faces two possibilities: leaving or staying and this last case – staying – encounter two choices: negotiating or staying without changes. In the following, we will consider the tenant's choices facing a break-option as:

- Leaving
- Staying
- Staying and negotiating

In Europe, the break-option is an asymmetric option in favour of the tenant. We investigate in our model the risk of tenants breaking their leases assuming a rational behaviour (ie: a tenant wants to increase its wealth or the wealth of its company) be it by reducing costs. Following Dupuy (2003), we build up a model which given a rationale behaviour is able to determine the choice of a tenant, incorporating it in the cash-flows. Our hypothesis of rational market players is capital here. All the tenants considered in our model are assumed to behave rationally and to use present value of outflows to make their decisions. Thereby, the tenants are supposed to act without sentiment and we do not consider strategic or political influence. Our model, instead of applying an arbitrary choice regarding the possibility of a tenant's leaving, will apply a pre-defined set of rules based on facts and observable and quantifiable events (such as the market rental value, the transaction costs, the rental value, the forecasted index and so on) and decide if the tenant stays (with possible negotiation) or leaves.

Facing a break-option, both the tenant and the landlord want to increase their wealth. The landlord wants to hedge its revenues and possibly to increase the value of its building and the tenant wants to minimize its expenses: the transaction or agency costs and the rent paid. Therefore, adopting a rational point of view and refusing all other considerations (availability of space, new business strategy, various influence), at the time of a break-option, the tenant will decide to stay in the premise; only if the rent currently paid is below the market rental value. The tenant will decide to leave the building if the price paid for the premises is higher

* In this paper, the term *transaction costs* regroup all the possible external costs induced by a moving: moving costs, broker fees, advisor fees, double rent, change of furniture, computer hardware change…

than the price they can find on the market (the market rental value). Possibly, the tenant will consider all the transaction costs incurred by a moving.

It is possible to find an analogy with traditional financial derivatives. The break-option works as a valuable option where premium can be determined, at least from a theoretical point of view. In finance, a European option is a contract between a buyer and a seller that gives the buyer the right but not the obligation to buy or sell a particular underlying asset at a later predefined date at an agreed price. In return for granting the option, the seller collects a premium from the buyer. In a capital market, a rational player will exercise a European option at the maturity as soon as it terminates in the money that is if its underlying price is above a predefined strike*. We remind here the value of a call written on a stock which pays a discrete dividend D based on the Black and Scholes model:

$$C_t = \max(S_T - De^{-r_f(T'-t)} - Ke^{-r_f(T-t)}; 0)$$

C_t is the value of the call at time t

r_f is the risk free

D is the dividend

T' is the distribution time of the dividend

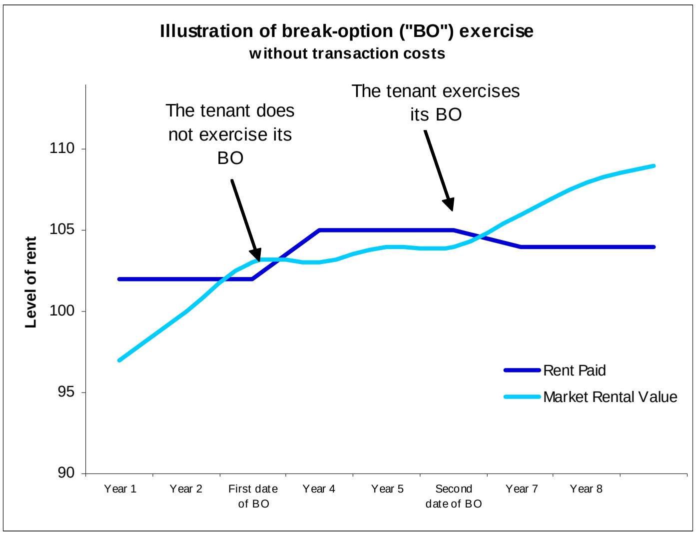
T is the maturity of the option

K is the strike of the option

S_t is the underlying asset at time t

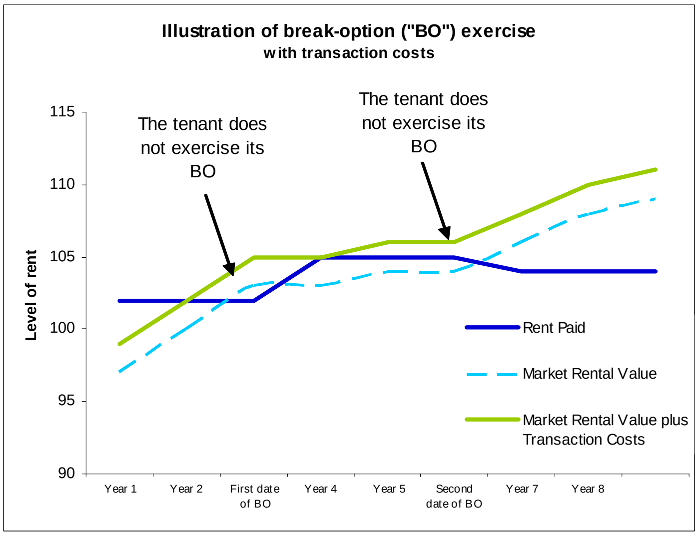
So, at the time of a break-option, the tenant has the right, but not the obligation, to terminate the lease. A rational tenant will thus terminate the lease only if its break-option is in the money and therefore only if the market rental value is below the rent paid. As said before, the tenant has two possibilities: staying or leaving. In terms of options: exercise or not exercise; and even so more: being in the money or not. This is illustrated by graph 2.1.

* A Call option is in the money if its underlying price is above a predefined price: St > K, in this case a rational player will exercise the call in order to buy the underlying asset at cheaper price.
A Call option is at the money if the price of the underlying asset it is written on, equal the strike price: St = K
A Call option is out of the money if its underlying price is below a predefined price: St < K



Graph 2.1 – Illustration of the tenant's behavior without transaction costs

It is essential to take the transaction costs into account in our analysis. They have a large impact on decisions. However they are quite difficult to assess. In fact the tenant will exercise its option to leave as soon as its rent is higher than the Market Rental Value plus the transaction costs. This is illustrated in the graph 2.2.



Graph 2.2 – Illustration of the tenant's behavior taking into account the transaction costs

The graph 2.2 illustrates how it is essential to take into account the transaction costs in our model. These transaction costs work as the discrete dividends of the financial option. In term of options, the break-option can thus be written as follow:

$$BO_t = \max(R_t - (MRV_t + T_c(t)); 0)$$

Consequently, the break option is a call on the rental value. Thus as soon as the tenant must rationally choose to stay or to leave, it can be analysed in terms of option (in the money vs out of the money). The rational player will thus decide to leave only if it is in the money.

Basically, for a lease and at the time of a break-option or at the end of the lease, we consider all the possibilities offered to a tenant; namely: leaving, staying or staying with a new negotiated rent. Under the assumption of rational behaviour, our model for each scenario determines which decision the tenant may take. To do so, we take into consideration the rent paid by the tenant at the time of the option, the simulated Market Rental Value and the transaction costs the tenant will face in case of moving. Indeed estimating the level of these three factors and comparing them, we determine the decision the tenant should take given a rational behaviour. Precisely, our methodology is to use the option's theory. Is the option it in the money? Using option's theory, our purpose is then only to determine if the option to leave will be exercised (we do not need to determine the premium of this option). If the option is in the money the tenant leaves, if not the tenant stays with a possible rent adjustment:

- In the case the option is in the money, the tenant vacates and the landlord has to find another tenant for a new price. Hypothetically, given the necessary time to find a new tenant and given the numerous advantageous financial conditions granted by the landlord given the state of the market (under pressure or not), a void in the cash-flow is applied each time an option to leave is exercised. Therefore each time an option is in the money, we incur a void in the cash flows corresponding to one period of time. After this, we consider that the landlord will sign a new lease with a new tenant for a new rent corresponding to the Market Rental Value. After that, the evolution of this new rent follows the index and another option could be granted by the landlord to the new tenant during the course of the simulation.

  Mathematically this case corresponds to:

$$R_t \geq MRV_t + T_c(t)$$

- In the case the option is out of the money the tenant wants to stay in the building. Two cases are possible:
  - The tenant's rent is below the market rental value: in this case the tenant stays in the building for the same rent until the end of the lease. The landlord is not in a strong position as he wants to be hedged against vacancy. So, no negotiation takes place during the course of the lease; eventually the rent is adjusted to the Market Rental Value if the considered option concerns the end of the lease.

    Mathematically this case corresponds to:

$$R_t \leq MRV_t$$

- The tenant's rent stands between the Market Rental Value and the Market Rental Value plus the transaction costs: in this case we consider both the tenant and the landlord adopt a rational behaviour and start negotiating. For simplification we consider they will concord to the market rental value.

Mathematically this case corresponds to:

$$MRV_t \leq R_t \leq MRV_t + T_c(t)$$

II.3. Combining options and Monte-Carlo simulations

Combining Monte-Carlo simulations with options conceded to the tenant is the innovating part of our model. Our proposition is to combine two models: that of Dupuy (2003) and that of Baroni, Barthélémy and Mokrane (2005). On one hand, we keep Baroni, Barthélémy and Mokrane's idea to simulate simultaneously the evolutions of the Price and of the Market Rental Value and on the other hand we keep Dupuy's idea to consider the options to leave granted to the tenant by considering the difference between the rent paid and the Market Rental Value (the possible rent the tenant might pay if it takes another space) available on the market. The tenants also consider the transaction costs they will face.

Using Monte-Carlo, a large number of scenarios are simulated – given any predefined distribution function – simultaneously for the Price and the Market Rental Value and compare for each scenario – at the relevant dates – the market rental value plus the transaction costs with the rents paid and rationally decide, for this specific scenario, if the option is exercised or not (if the option is in the money or not).

Our model seeks to value real estate asset or real estate portfolio considering the risk inherent in the cash flow by taking into account the numerous lease structures that exist in a portfolio or simply in a multi-let asset. From each scenario of the Monte-Carlo simulation (where we simulate simultaneously a market rental value path, a price path and consider a rent path), we will obtain a value for the asset or for the portfolio. To compute this value, the model simply determines all the future cash flows received by the owner of the portfolio which are the rent and the final price and discount it. We incorporate the risk inherent in the lease structure in the cash flows.

Mathematically, for a lease sign for term *T-1* and with a break option at the time $t < T-1$ and for a simulation horizon of *T+1*, our model can be implemented to get the price of the asset using the next formula. In this example, we have considered that as soon as a tenant leaves, the landlord suffer from a lack in cash flows equivalent to one period. In addition, we only consider for simplification in the presentation one lease structure, so, as soon as a tenant leaves, the following tenant signs under the same lease structure conditions.

We consider the three possibilities offered to the tenant: leaving, negotiating and staying. In the following formula, these possibilities appear according to the next order:

$$K = \begin{cases} K_1 = leaving \\ K_2 = negotiating \\ K_3 = staying\ in\ the\ building\ without\ change \end{cases}$$

For one Monte-Carlo scenario, the Price computed corresponds to the following formula where each brace represents the period following a break option:

$$P_0 = \frac{CF_0 \times (1+I_0)}{(1+k)} + \sum_{i=2}^{t-2} \frac{CF_0 \times \prod_{j=0}^{i-1}(1+I_j)}{(1+k)^i} + \frac{CF_0 \times (1+I_0) \times \cdots \times (1+I_{t-2})}{(1+k)^{t-1}} + K + \frac{P_{T+1}}{(1+k)^{T+1}}$$

With K as follows:

$$K = \begin{cases}
K_1 = 0 + \frac{MRV_{t+1}}{(1+k)^{t+1}} + \cdots + \frac{MRV_{t+1} \times (1+I_{t+2}) \times \cdots \times (1+I_{2t})}{(1+k)^{2t}} + \begin{cases} 0 + \frac{MRV_{2t+2}}{(1+k)^{2t+2}} + \cdots + \frac{MRV_{2t} \times (1+I_{2t+2}) \times \cdots \times (1+I_T)}{(1+k)^{T+1}} \\ \frac{MRV_{2t+1}}{(1+k)^{2t+1}} + \cdots + \frac{MRV_{t+1} \times (1+I_{2t+1}) \times \cdots \times (1+I_T)}{(1+k)^{T+1}} \\ \frac{MRV_{t+1} \times (1+I_{t+2}) \times \cdots \times (1+I_{2t})}{(1+k)^{2t+1}} + \cdots + \frac{MRV_{t+1} \times (1+I_{2t+1}) \times \cdots \times (1+I_T)}{(1+k)^{T+1}} \end{cases} \\
K_2 = \frac{MRV_t}{(1+k)^t} + \cdots + \frac{MRV_t \times (1+I_t) \times \cdots \times (1+I_{T-2})}{(1+k)^{T-1}} + \begin{cases} 0 + \frac{MRV_{T+1}}{(1+k)^{T+1}} \\ \frac{MRV_T}{(1+k)^T} + \frac{MRV_T \times (1+I_T)}{(1+k)^{T+1}} \\ \frac{MRV_t \times (1+I_t) \times \cdots \times (1+I_{T-1})}{(1+k)^T} + \frac{MRV_t \times (1+I_t) \times \cdots \times (1+I_T)}{(1+k)^{T+1}} \end{cases} \\
K_3 = \frac{CF_0 \times (1+I_0) \times \cdots \times (1+I_{t-1})}{(1+k)^t} + \cdots + \frac{CF_0 \times (1+I_0) \times \cdots \times (1+I_{T-2})}{(1+k)^{T-1}} + \begin{cases} 0 + \frac{MRV_{T+1}}{(1+k)^{T+1}} \\ \frac{MRV_T}{(1+k)^T} + \frac{MRV_T \times (1+I_T)}{(1+k)^{T+1}} \\ \frac{CF_0 \times (1+I_0) \times \cdots \times (1+I_{T-1})}{(1+k)^T} + \frac{CF_0 \times (1+I_0) \times \cdots \times (1+I_T)}{(1+k)^{T+1}} \end{cases}
\end{cases}$$

I_j corresponds to the index for the period j*. $I_0 = 0$

CF_i to the cash flow i

k is the discount rate

MRV_i is the Market Rental Value at time i

P_i is the Price at time i

* Indeed, the period *j+1* is indexed on the index of the period j. Indexes are traditionally published a quarter after the end of their respective period.

III. 1. Application of the model

In this section, we will analyze and show how our model can be used by investors to maximize but also to analyze the value of a Portfolio taking into account the risk associated to the lease structure. In particular, we underline the robustness of the model.

To do so, we compute the value and the inflows of a given portfolio along 15 years for each year. We will analyze various scenarios regarding the principal inputs of the model which are:

- The lease structure
- The trends of the Market Rental Values
- The trends of the Prices
- The indexation of the rents
- The volatilities of the Market Rental Values
- The volatilities of the Price
- The transaction costs

We particularly focus on the parameters which impact most strongly the tenant's behavior and the valuation. These parameters are the lease structure, the transaction costs, the rents indexation and the trend of the Market Rental Values.

To simplify the presentation of the model and its results, we consider an example of a portfolio composed of assets which are located in the same market. Therefore we consider only one tendency for the market rental values we will simulate. However the leases specificities are considered.

For our implementation of the model, we make the following assumptions:

- The correlation between the Price and the numerous Market Rental Values is supposed to be fixed along the time and equal to 60%. This correlation is the same for all the market rental value as they all evolve in the same sub-market.
- The transaction costs are supposed to be depreciated. The length of depreciation is a parameter of the model. This depreciation is relevant as it translates both the accounting reality and the fact that the tenants are more tempted to move after a long time in the building.
- For each lease, the structure remains the same along the time, even if the tenant vacates (the new tenant signs a contract with the previous lease structure).
- The volatility is determined for each lease (each sub-market) and is constant during the time.
- The void period is fixed to one year.
- The discount rate is constant during the time and fixed to 6,5%.
- The portfolio is acquired in 2009 and its composition is supposed to be unchanged during the analysis period. No arbitrages are possible.
- The tenants are rational.

- To simplify the readability of the results, we consider only yearly cash flows. Furthermore, in our implementation, all the cash flows happen at the same date.
- In addition, we consider our portfolio evolves in a tax-exempt area and is not leveraged (100% equity).

The Portfolio we propose to analyze is constituted of four assets and six tenants. All the assets are fully at the beginning of the simulation. Therefore six leases compose the portfolio. The initial Price is equal to 100 M€.
Leases 1 and 2 correspond to the first asset.
Leases 3 and 4 correspond to the second and the third asset.
Leases 5 and 6 correspond to the fourth asset.
We thus analyze four sub-markets.

The Market Rental Values the spaces worth and the rents produced by the various tenants are displayed in the table 3.1. As shown in this table, two spaces are let at their rental values, three spaces are somehow over-rented and one space is below its market rental value. The leases 2 and 3 are subject to a reversion possibility of respectively 17 and 20 %, therefore these two leases bear a high break risk. The lease 4 is let under the market rental value it worth; the tenant might thereby be tempted to stay in the premise. This situation of a portfolio globally let over the market rental value translate the case observed in Europe in the beginning of 2010. Numerous rents have been positively indexed during the last five years when the market rental values have shown a large fall.

in M€ per year	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
Initial Market Rental Value	1,50	1,00	1,20	1,30	0,80	0,70
Initial Rent	1,50	1,20	1,50	1,20	0,80	0,80
Difference btw rent & MRV	**0%**	**-17%**	**-20%**	**8%**	**0%**	**-13%**

Table 3.1 – Market Rental Value and Rents of the spaces that composed the portfolio

The total Portfolio produce initially a rent of 7,0 M€ and have a market rental value equal to 6,5 M€. Thus the potential immediate negative reversion is equal to 7%. This is presented in the table 3.2.

Total Portfolio = 100 M€	
MRV of all the leases	6,50
Rents Produced	7,00
Potential reversion	**-7%**

Table 3.2 – Overall Market Rental Value and Rent of the Portfolio

The lease structure of all the contracts is presented in the following table 3.3. The beginning of the period starts in 2010; therefore the portfolio was bought one year before. Overall during the next three years (until 2013), four leases (lease 2, 3, 4 & 5) representing 4.3M€ of inflows are at risk. Globally the repartition of the risky dates is not that concentrated for this portfolio.

Lease structure	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
First date of break option	2 016	2 012	2 013	2 011	-	2 014
Second date of break option	-	2 015	-	2 016	-	-
End of lease	**2 019**	**2 018**	**2 018**	**2 021**	**2 012**	**2 017**

Table 3.3 – Break-option and end of lease of all the leases

The table 3.4 presents the transaction costs of each lease and their depreciation time. It is capital to estimate the level of transaction costs a tenant will face. Basically the higher the transactions costs are, the higher the time spent in the premises may be, as the tenants do not want to face these costs too often.

Transaction costs in M€	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
Direct transaction costs	1,0	0,2	1,0	0,5	-	0,4
Depreciation time	8	6	10	30	1	7

Table 3.4 – Transaction costs and their depreciation periods for each lease

The trend and the volatility of the market rental values and the rents indexation are presented in table 3.5. The market rental values' trend reflects a potential fall over the next two years and then a recovery of the level of the market rental values for the remaining simulation horizon. Given the specificities of the lease, we assume different indexation possibilities The lease 2, 5 and 6 are indexed on the same index, the lease 4 is indexed on 80% of the index. The lease 2 is capped at 3.5% and uses the same index and the lease 3 grow of 1.5% per year. Globally, the indexation is positive over the course of the simulation and range from 1 to 4%. The MRV's volatility is relatively low and fixed at 8%.

Years	MRV	Lease 1 to 6	Rent	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
2010	MRV's Trend	-3,0%	Rent's indexation	1,5%	1,5%	1,5%	1,2%	1,5%	1,5%
2011		-1,0%		1,0%	1,0%	1,5%	0,8%	1,0%	1,0%
2012		0,0%		1,2%	1,2%	1,5%	1,0%	1,2%	1,2%
2013		1,0%		3,7%	3,5%	1,5%	3,0%	3,7%	3,7%
2014		2,0%		4,0%	3,5%	1,5%	3,2%	4,0%	4,0%
2015		2,5%		3,8%	3,5%	1,5%	3,0%	3,8%	3,8%
2016		3,0%		3,5%	3,5%	1,5%	2,8%	3,5%	3,5%
2017		3,5%		3,6%	3,5%	1,5%	2,9%	3,6%	3,6%
2018		2,8%		3,1%	3,1%	1,5%	2,5%	3,1%	3,1%
2019		2,3%		2,6%	2,6%	1,5%	2,0%	2,6%	2,6%
2020		2,0%		2,2%	2,2%	1,5%	1,8%	2,2%	2,2%
2021		2,0%		1,8%	1,8%	1,5%	1,4%	1,8%	1,8%
2022		1,5%		1,8%	1,8%	1,5%	1,4%	1,8%	1,8%
2023		1,5%		1,8%	1,8%	1,5%	1,4%	1,8%	1,8%
2024		1,5%		1,8%	1,8%	1,5%	1,4%	1,8%	1,8%
MRV's Volatility:		8%							

Table 3.5 – Market Rental Values trends and volatilities and rents indexation per lease

The Price's parameters are displayed in the graph 3.6. The volatility is supposed to be constant over time and relatively low to 6%. Furthermore, we assume a rather low evolution of the price in the future in order to keep a conservative point of view.

Years	Price trend	Price volatility
2010	2%	6%
2011	2%	6%
2012	2%	6%
2013	3%	6%
2014	3%	6%
2015	3%	6%
2016	3%	6%
2017	2%	6%
2018	2%	6%
2019	2%	6%
2020	2%	6%
2021	2%	6%
2022	2%	6%
2023	2%	6%
2024	2%	6%

Table 3.6 – Price trends and volatilities

All these inputs parameters form the base case of our example. In the following we modify successively one parameter to underline how our model takes it into account. This underlines the sensitivities of our model to the various inputs and the efficiency of our model.

The table 3.7 shows the results of the simulation for the base case we have just introduced. These results form the basis of our comparison. The table 3.8 presents the probability of vacancy for the base case.

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Final value of the Portfolio	**100**	**102**	**104**	**105**	**107**	**109**	**111**	**112**	**113**	**114**	**115**	**116**	**116**	**117**	**117**	**118**
Simulated Price of the Portfolio	100	101	104	106	108	111	114	117	119	121	124	126	129	131	134	137
Discounted Price of the Portfolio	100	95	91	87	84	81	78	75	72	69	66	63	60	58	55	53
Rents indexed	7,0	7,1	7,2	7,3	7,5	7,7	8,0	8,2	8,5	8,7	8,9	9,1	9,2	9,4	9,5	9,7
Rents produced	7,0	7,1	7,2	5,6	7,1	6,8	7,1	6,6	7,1	6,3	6,8	7,5	6,5	7,3	7,4	7,6
aggregated Simulated MRV	6,5	6,3	6,2	6,2	6,3	6,4	6,6	6,8	7,0	7,2	7,4	7,5	7,7	7,8	7,9	8,0

Table 3.7 – Results of the simulation for the base Case

	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
2009	0%	0%	0%	0%	0%	0%
2010	0%	0%	0%	0%	0%	0%
2011	0%	0%	0%	0%	0%	0%
2012	0%	89%	0%	0%	73%	0%
2013	0%	0%	3%	0%	0%	0%
2014	0%	0%	0%	0%	0%	56%
2015	0%	25%	0%	0%	0%	0%
2016	50%	0%	0%	6%	0%	0%
2017	0%	0%	0%	0%	0%	36%
2018	0%	3%	73%	0%	0%	0%
2019	28%	0%	0%	0%	0%	4%
2020	0%	0%	0%	0%	0%	0%
2021	0%	53%	6%	18%	0%	1%
2022	0%	0%	0%	0%	0%	29%
2023	11%	0%	0%	0%	0%	0%
2024	0%	1%	0%	0%	0%	0%

Table 3.8 – Probability of vacancy of the base case

These results are interesting as they show that the three over-rented leases (leases 2, 3 and 6) rapidly present a high probability of vacation, are quickly vacated depending of the level of transaction costs. The leases 1 and 5 which are let at their market price show a different case: the premises are not almost certainly vacated during the first years of the simulation. This observation highlights how an asset let at its market price is less risky than an asset which generates more cash flows but at a rent higher than its market rental value. Furthermore, our model proves to be a good practical tool for asset management. A landlord aware of the risks he faces can anticipate and in some cases prevent the future changes.

III. 2. Sensitivity analysis

III. 2. 1 Lease structure

In this section we show how our model takes into account the impact of the lease structures and of the allocation of the break-option dates on the value of a portfolio. Firstly, we present a case where all the contracts have the same maturity and secondly we present a case where the tenants can break their lease each year.

III.2.1.1. Tenants with the same lease structure

We present in the table 3.9 the assumptions we change for the first case. All the leases can be broken in 2013 and 2016 and end in 2019. All the other parameters equal the base case scenario.

Lease structure	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
First date of break option	2 013	2 013	2 013	2 013	2 013	2 013
Second date of break option	2 016	2 016	2 016	2 016	2 016	2 016
End of lease	**2 019**	**2 019**	**2 019**	**2 019**	**2 019**	**2 019**

Table 3.9 – Modified lease structure of all the leases for the first case

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Final value of the Portfolio	**100**	**102**	**104**	**106**	**107**	**109**	**112**	**112**	**112**	**113**	**114**	**115**	**115**	**116**	**116**	**117**
Simulated Price of the Portfolio	100	102	104	106	108	111	114	117	119	121	124	126	129	131	134	137
Discounted Price of the Portfolio	100	95	91	87	84	81	78	75	72	69	66	63	60	58	55	53
Rents indexed	7,0	7,1	7,2	7,3	7,5	7,7	8,0	8,2	8,5	8,7	8,9	9,1	9,2	9,4	9,5	9,7
Rents produced	7,0	7,1	7,2	7,3	5,4	7,2	7,4	4,9	6,4	7,2	6,0	7,1	7,3	7,4	6,5	7,4
aggregated Simulated MRV	6,5	6,3	6,2	6,2	6,3	6,4	6,6	6,8	7,0	7,2	7,4	7,5	7,7	7,8	7,9	8,0

Table 3.10 – Results of the simulation with the leases having all the same lease structure

The results show a small decrease in the value of the portfolio. Focusing on the rents produced, we observe a large fall in the quantity of cash corresponding to the year 2013 and to the year 2016 where the tenant can vacate the premises.

III. 2.1.2. Tenants with a possibility to leave each year

In table 3.11 are presented the assumptions. The tenants can vacate the premise each year.

Lease structure	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
First date of break option	2 011	2 011	2 011	2 011	2 011	2 011
Second date of break option	2 012	2 012	2 012	2 012	2 012	2 012
Third date of break option	2 015	2 015	2 015	2 015	2 015	2 015
Fourth date of break option	2 014	2 014	2 014	2 014	2 014	2 014
Fifth date of break option	2 015	2 015	2 015	2 015	2 015	2 015
End of lease	**2 016**	**2 016**	**2 016**	**2 016**	**2 016**	**2 016**

Table 3.11 – Modified lease structure – The tenants can terminate their lease each year.

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Final value of the Portfolio	**100**	**102**	**103**	**105**	**107**	**108**	**109**	**110**	**110**	**111**	**112**	**112**	**113**	**113**	**114**	**114**
Simulated Price of the Portfolio	100	102	104	106	108	111	114	117	119	121	124	126	129	131	134	137
Discounted Price of the Portfolio	100	95	91	87	84	81	78	75	72	69	66	63	60	58	55	53
Rents indexed	7,0	7,1	7,2	7,3	7,5	7,7	8,0	8,2	8,5	8,7	8,9	9,1	9,2	9,4	9,5	9,7
Rents produced	7,0	7,1	5,6	6,7	6,8	6,3	6,2	5,4	6,5	6,5	6,9	6,6	6,9	7,0	7,0	7,3
aggregated Simulated MRV	6,5	6,3	6,2	6,2	6,3	6,4	6,6	6,8	7,0	7,2	7,4	7,5	7,7	7,8	7,9	8,0

Table 3.12 – Results of the simulation with the tenants having the possibility to vacate each year

Once more the value of the portfolio decreases. This is due to the decline in the quantity of inflows produced. The value of the portfolio, on the contrary of the simulated price, is the sum of all the discounted cash flows of the model plus the discounted simulated price. The simulated price is the final price estimated by our model. It does not take into account any cash flows. The graph 3.12 shows a somehow larger diminution of the value of the portfolio but underlines particularly how an investor interested in cash on cash and not in capital gain could be disoriented. This is due to the rents produced which are clearly below those of the base case.

In particular, these two cases show how our model can be use to analyze the cash on cash distribution how it make it possible to correctly take into account the lease structure risk.

III.2.2. The transaction costs

In this part, we analyze the effect of the transaction costs on tenant's behavior and show how our model takes this input into account. Intuitively, large transaction costs for the tenant induce longer time spent in the premises because tenants want to avoid these costs as often as possible. Firstly we present a case tenants do not face transaction costs and secondly we present a case where the transaction costs are rather high.

III.2.2.1. No transaction costs

The table 3.13 shows the transaction costs faced by the tenants in this first case. In this case the tenants do not face transaction costs and just analyze the price paid for the premise. This case happens in a bear market when the landlords accept to pay the moving costs and to offer attractive financial conditions to attract tenants in their assets.

Transaction costs in M€	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
Direct transaction costs	-	-	-	-	-	-
Depreciation time	-	-	-	-	-	-

Table 3.13 – Tenants facing no transaction costs

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Final value of the Portfolio	**100**	**102**	**104**	**105**	**105**	**107**	**108**	**108**	**109**	**110**	**110**	**111**	**111**	**112**	**112**	**113**
Simulated Price of the Portfolio	100	102	104	106	108	111	114	117	119	121	124	126	129	131	134	137
Discounted Price of the Portfolio	100	95	91	87	84	81	78	75	72	69	66	63	60	58	55	53
Rents indexed	7,0	7,1	7,2	7,3	7,5	7,7	8,0	8,2	8,5	8,7	8,9	9,1	9,2	9,4	9,5	9,7
Rents produced	7,0	7,1	6,8	5,4	5,3	6,0	6,2	5,1	6,2	6,8	6,6	7,1	6,6	7,0	6,6	7,4
aggregated Simulated MRV	6,5	6,3	6,2	6,2	6,3	6,4	6,6	6,8	7,0	7,2	7,4	7,5	7,7	7,8	7,9	8,0

Table 3.14 – Results of the simulation when tenants do not face transaction costs

In this case the value of the portfolio is brought down. This is due to the rents produced. The tenants leave their premises as soon as at the time of a break-option, the market rental value is below the rent paid. Therefore the tenants leave more often than usual and the landlords face numerous void time.

III.2.2.2. Large transaction costs (3 times the annual rent) and a long depreciation period

Transaction costs in M€	Lease 1	Lease 2	Lease 3	Lease 4	Lease 5	Lease 6
Direct transaction costs	4,5	3,6	4,5	3,6	2,4	2,4
Depreciation time	10	10	10	10	10	10

Table 3.15 – Tenants facing really high transaction costs (3 times their rent)

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Final value of the Portfolio	**100**	**102**	**104**	**106**	**109**	**111**	**114**	**116**	**118**	**119**	**120**	**120**	**120**	**121**	**122**	**122**
Simulated Price of the Portfolio	100	101	104	106	108	111	114	117	119	121	124	126	129	131	134	136
Discounted Price of the Portfolio	100	95	91	87	84	81	78	75	72	69	66	63	60	58	55	53
Rents indexed	7,0	7,1	7,2	7,3	7,5	7,7	8,0	8,2	8,5	8,7	8,9	9,1	9,2	9,4	9,5	9,7
Rents produced	7,0	7,1	7,2	7,2	7,4	7,7	7,9	8,1	8,1	7,4	6,9	7,0	5,7	7,4	7,4	7,7
aggregated Simulated MRV	6,5	6,3	6,2	6,2	6,3	6,4	6,6	6,8	7,0	7,2	7,4	7,5	7,7	7,8	7,9	8,0

Table 3.16 – Results of the simulation in the case the tenant face large transaction costs

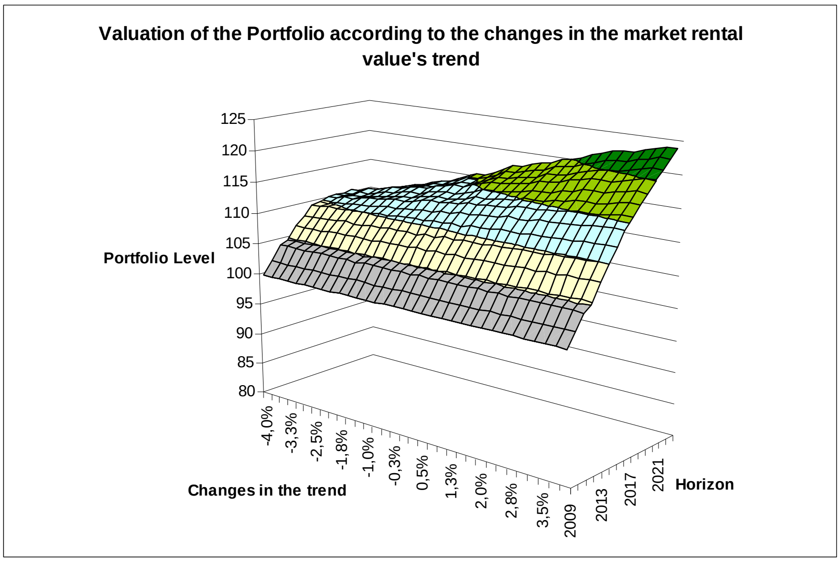
As shown in the table 3.16, the value of the portfolio increases when the tenants face large transaction costs. Even if they face higher than usual rent, the tenants are tempted to stay in the premises in order to avoid these costs.

With these two previous cases, we show the large impact of the transactions cost on the tenant's behaviour and the consequences on the portfolio value. Including the depreciation of the transactions costs we are able to get a more precise measurement of this impact.

III.2.3. Trend of the Market Rental Value

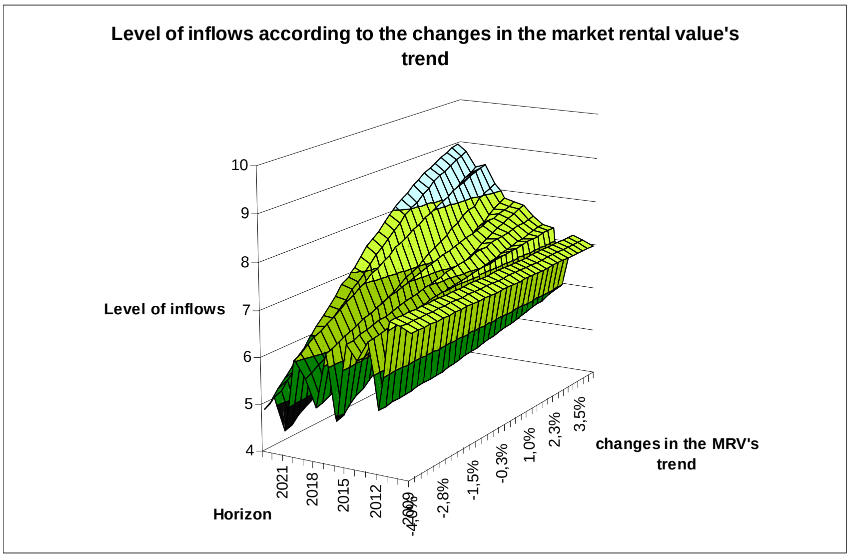
In this third section, we analyze the sensitivity of our model to a change in the trend estimation. Precisely we move all the trends from -4% to +4% from their initial values; this means that the trend of 2% expected in 2014 ranges in our simulations from -2% to 6%. The graph 3.1 presents the change in the value of the portfolio according to changes in the MRV's trend and graph 3.2 presents the level of cash flow according to changes in MRV's trend.

The results of the graph 3.1 point out how trends changes transform the value of the portfolio particularly over long horizon when the weight of the terminal value decreases. It shows the value of the portfolio is capped when the MRV's trend is negative. In this case the investor has no interest to keep the portfolio on a long run.



Graph 3.1 – Value of the Portfolio according to changes in the MRV's trend

The graph 3.2 focuses on the inflows. It underlines how the level of cash flows is modified by the tenant's behavior and how our model takes into account the tenant's behavior according to the lease structure. The proposed approach is therefore particularly relevant to analyze the revenues. When leveraging a portfolio, an operator should be interested by studying the level of its revenues in order to be able to cover all its debt service expenses or simply the determine the optimal level of debt.



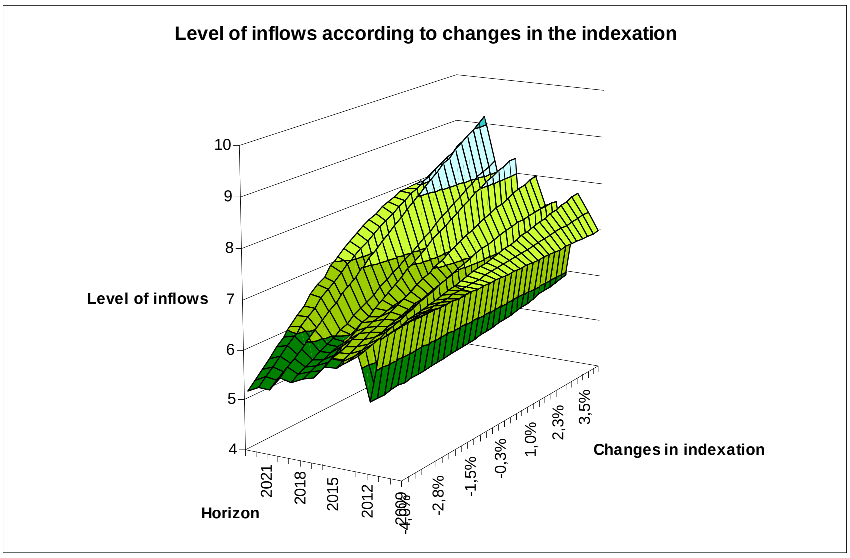
Graph 3.2 – Level of cash flows according to changes in the MRV's trend

A threshold in the revenues appears in some years and could be useful to compute the acceptable leverage.

III.2.4. Rents indexation

In this section we analyse the sensitivity of the cash flows to changes in the rent indexation.

Graph 3.3 shows how our model incorporates the cash flows risk for a predefined lease structure. As expected, the level of inflows increases when the index itself increases. However, we can notice that when the indexation is higher these cash flows are much more volatile and impacted by the lease structure.



Graph 3.3 – Level of cash flows according to changes in the rent's indexation

As for the previous sections, the model incorporates the risk due to the lease structure and is able to measure it according to the rent indexation.

Conclusion

In this paper, we have considered Monte-Carlo methodology and options to compute the price of a real estate portfolio. In doing so, we incorporate risk in the valuation process and we especially take into account the risk linked to the lease structure.

Our model presents numerous advantages with respect to other traditional valuation models; in particular, following Baroni, Barthélémy and Mokrane (2006), the terminal value is simulated and not determined by cash flow capitalization based on infinite growth rate. In addition, following Dupuy (2003) the cash flows incorporate the tenants' departure risk taking into account the state of the market. Furthermore our model is more efficient for the measurement of both complex cash generating assets and terminal value. Through the use of sensitivity analysis, we have demonstrated how this modeling can help the investor to measure risk on the portfolio value and to adopt a leverage ratio consistent with the future cash flows.

This research reveals a path for a plethora of other possible applications. Amongst which, one can imagine to combine our model with American option theory in order to derive an optimal holding period that can capture the dynamics of the properties that compose the portfolio. Another application idea could be to derive a Credit-VaR from our model by leveraging the portfolio and considering the default risk.

Our main conclusions are that our model allows a finer analysis of the cash flows than traditional model and makes one able to take into account the risk inherent in the lease structure, the transaction costs, the evolution of the rental market and the rent indexation.

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