Inflation-Hedging, Asset Allocation, and the Investment Horizon

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Abstract

Focusing on the role of the investment horizon, we analyze the inflation-hedging abilities of stocks, bonds, cash and direct commercial real estate investments. Based on vector autoregressions for the UK market we find that the inflation-hedging abilities of all assets improve with the investment horizon. For long horizons, real estate seems to hedge unexpected inflation as well as cash. This has implications for the difference between the return volatility of real returns versus the return volatility of nominal returns, and ultimately for portfolio choice. Portfolio optimizations based on real returns yield higher allocations to cash and real estate than optimizations based on nominal returns. Bonds tend to be less attractive for an investor taking into account inflation. Switching from nominal to real returns, the allocation to stocks is decreasing at medium investment horizon, but increasing at long horizons.

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1. Introduction

The monetary base has grown considerably in many economies as a reaction to the current financial crisis. As a result, the fear of inflation has regained attention. Even modest inflation rates can have a significant effect on the real value of assets when the investment horizon is long. For example, €100 invested for 20 years at a nominal interest rate of 5% p.a. yield €265.3 final wealth, compared to €180.6 assuming a real annual interest rate of 3%. Hence, an inflation rate of $1.05/1.03-1\approx 1.94\%$ p.a. reduces the real value of the investment by 32%. Despite the important role of inflation for decision making, people often think in nominal rather than real terms, a phenomenon referred to as “money illusion” (for a review see Akerlof and Shiller 2009, Chapter 4). Assets that hedge inflation are desirable for private investors concerned about the purchasing power of their investments as well as for institutional investors whose liabilities are linked to inflation (such as pension funds).

Most of the evidence on optimal portfolio choice is based on the traditional Markowitz (1952) approach with quarterly or annual returns used to estimate expected returns, standard deviations, and correlations. This common procedure contrasts with the fact that most investors have longer investment horizons. Due to return predictability, standard deviations (per period) and correlations of asset returns may change considerably with the investment horizon (Campbell and Viceira 2005). Hence, the optimal asset allocation depends on the investment horizon. The asset classes usually considered for a mixed asset allocation optimization are cash, bonds and stocks. Real estate is a further important asset class. In the US, for example, the market capitalization of private commercial real estate is estimated to be $8 trillion, compared to a value of $17 trillion for stocks, as of the early 2000s (Geltner et al. 2007, Chapter 7). Due to high transaction costs, there are substantial horizon effects in periodic expected returns on real estate (e.g., Collet et al. 2003). This is certainly a reason why direct real estate investments are typically long-term investments with an average holding period of about ten years (Collet et al. 2003, Fisher and Young 2000). Practitioners often regard direct real estate investments to be a good inflation hedge.

In this paper, we link the inflation-hedging analysis with the mixed asset allocation analysis, focusing on the role of the investment horizon for a buy-and-hold investor. Using a vector autoregression (VAR) for the UK market, we estimate correlations of nominal returns with inflation, analyzing how the inflation hedging abilities of cash, bonds, stocks and direct commercial real estate change with the investment horizon.[1] The results have implications for the difference between the term structures of annualized volatilities of real versus nominal returns, and ultimately for portfolio choice. The differences in the optimal asset weights (based on real versus nominal returns) can be interpreted as the mistake that an investor subject to inflation illusion makes. On the other hand, the results for nominal rather than real returns are relevant for investors facing liabilities that are fixed in nominal terms.

The remainder of the paper is organized as follows: In the next section, we will review the related literature. A discussion of the VAR model, the data and the VAR results follow. Then, we will analyze horizon effects in risk and return for nominal and real returns. In this section, the results with regard to the inflation hedging abilities of the assets will also be discussed. The asset allocation problem will be examined in the next section, again distinguishing between nominal and real returns. A discussion of a robustness check follows. Finally, the main findings will be summarized.

2. Literature review

Academics have devoted much attention to the abilities of assets to hedge inflation. Bodie (1976), Jaffe and Mandelker (1976) and Fama and Schwert (1977) find that that nominal US stock returns are negatively related to realized inflation as well as to the two components of realized inflation, i.e., expected and unexpected inflation. Gultekin (1983) shows that the negative relation of nominal stock returns with inflation also holds for many other countries. The perverse inflation-hedging characteristics of stocks run contradictory to the general belief that stocks should be a good hedge against inflation due to the fact that stocks are essentially claims to cash-flows derived from real assets. Of all the US assets examined by Fama and Schwert (1977) (government bills and bonds, residential real estate, human capital and stocks), residential real estate is the only asset that provides a complete hedge against inflation. (An asset is said to be a complete hedge against inflation when the coefficients from a regression of nominal returns on proxies for expected and unexpected inflation are both statistically indistinguishable from one.) Bonds and bills provide a hedge against expected inflation, but not against unexpected inflation. Studies examining the direct commercial real

[1] Inflation-linked bonds with a maturity equal to the investment horizon are a particularly good inflation-hedge. Given the limited supply of these bonds, it is worthwhile to analyze the inflation-hedging abilities of common asset classes.

estate market suggest at least a partial inflation hedge. US commercial real estate appears to offer a hedge against expected inflation, whereas the evidence with regard to unexpected inflation is not clear-cut (e.g., Brueggeman et al. 1984, Hartzell et al. 1987, Gyourko and Linneman 1988, Rubens et al. 1989). Examining the UK market, Limmack and Ward (1987) find that commercial real estate returns are positively related to both expected and unexpected inflation. Depending on the proxy for expected inflation, however, commercial real estate does not appear to provide a hedge against both components. In contrast to the direct real estate market, real estate stocks tend to be negatively related or unrelated to expected and unexpected inflation (e.g., Liu et al. 1997, and Maurer and Sebastian 2002).

The results of the above-cited studies are based on regressions with data that have a monthly to annual frequency. The disappointing short-term inflation hedging abilities of most asset classes motivated research analyzing the long-term relation of asset returns with inflation. For both the US and the UK, Boudoukh and Richardson (1993) find positive relationships between five-year stock returns and realized as well as expected inflation, whereas annual returns show a negative or only weakly positive relationship. Based on cross-sectional regressions for 14 countries using data over a 14-year period, Quan and Titman (1999) find evidence that real estate is a hedge against realized inflation in the long run. In contrast, time-series regressions suggest that annual returns do not hedge against realized inflation. Hoesli et al. (2007) as well as Schätz and Sebastian (2009) use error correction approaches to distinguish between short- and long-term relationships between asset markets and macroeconomic variables. Hoesli et al. analyze the inflation hedging abilities of stocks as well as direct and securitized real estate markets in the US and the UK. For all asset markets, they find a positive long-term relationship with expected inflation. The long-term link to unexpected inflation is negative for all US assets and for UK stocks. UK property shares and direct real estate are positively linked to unexpected inflation in the long-run. In both countries, asset returns adjust rather slowly towards the long-term equilibrium, though. Schätz and Sebastian find a positive long-term link between commercial real estate markets and price indexes for both the UK and Germany. Confirming the findings of Hoesli et al., they observe that property markets in both countries are sluggish to adjust towards the long-term equilibrium existing with macroeconomic variables.

Several articles use a vector autoregressive (VAR) approach to estimate horizon-dependent correlation statistics. As the predictability of the variables is taken into account, the inflation hedging abilities of the assets are analyzed in terms of the correlation of unexpected asset returns with unexpected inflation. Campbell and Viceira (2005) calculate correlations of inflation shocks with unexpected *real* US stock returns. The correlation turns from weakly negative at short horizons to substantially negative at intermediate horizons, but it is slightly positive at the 50-year horizon. Hence, with the real return being almost unaffected by inflation, stocks seem to hedge unexpected inflation in the very long run. Hoevenaars et al. (2008) calculate correlations between unexpected *nominal* US asset returns and inflation shocks for horizons of up to 25 years. They find that cash is clearly the best inflation hedge for investment horizons of one year and longer. Bonds are a perverse inflation hedge in the short run; the correlation turns positive after about 12 years to reach more than 0.5 after 25 years. The correlation of nominal stock returns and REIT returns with inflation is negative in the short and slightly positive in the long run. Amenc et al. (2009a) report similar results with regard to cash and stocks. However, the estimated correlation of nominal REIT returns with inflation is about zero and the correlation of bonds is negative for all investment horizons. While the empirical evidence is not unambiguous, the general picture that emerges is that the inflation-hedging abilities of assets improve with the investment horizon.

Of course, the different inflation hedging characteristics of the assets have portfolio implications. Intuitively, when the nominal return on an asset is highly positively correlated with inflation, this decreases the volatility of real returns on the asset. Hence, the better the inflation-hedging ability of the asset, the more attractive is it for an investor concerned about real returns. Schotman and Schweizer (2000) show that when the investor is concern about real returns, the demand for stocks in a portfolio with a nominal zero-bond (with a maturity that equals the investment horizon) depends on two terms. The first term reflects the demand due to the equity premium. The second term depends positively on the covariance of nominal stock returns with inflation and represents the inflation hedging demand. The hedging demand changes with the investment horizon; depending on the parameterization of the model, the long-term hedging demand might be negative or positive.

Several articles calculate horizon-dependent risk statistics and optimal portfolio compositions based on real returns. Campbell and Viceira (2005) show that return predictability induces major horizon effects in annualized standard deviations and correlations of real US stock, bond and cash returns. Stocks exhibit mean reversion such that the periodic long-term volatility of real returns is only about 50% of the short-term volatility. Bonds exhibit slight mean reversion, whereas cash returns are mean averting. There are huge horizon effects in optimal portfolio compositions. In addition to stocks, bonds and cash investments, Fugazza et al. (2007) consider European property shares, whereas MacKinnon and Al Zaman (2009) consider US direct real estate and REITs.[2]

Analyzing the UK market, we follow the studies using a VAR approach. Hoevenaars et al. (2008) and Amenc et al. (2009a) analyze the US market including securitized real estate as an asset class, whereas we look at the UK market and focus on direct real estate. Hoevenaars et al. emphasize that the dynamics of REIT returns are well captured by the dynamics of stock and bond returns, so that the opportunity to invest in securitized real estate does not add much value for the investor. Given the different market microstructures of the securitized and the direct real estate market and the effect of leverage on the returns of securitized real estate, among other differences, it is interesting to analyze the direct real estate market. In addition, the market capitalization of direct commercial real estate still far exceeds the market capitalization of property shares in the UK (as in many other countries). As of the end of 2008, the market capitalization of the investable direct commercial real estate market is estimated to be about €250 billion, compared to a market capitalization of €64 billion for listed real estate companies.[3] Given the huge importance of transaction costs for direct real estate investments, we account for the differing transaction costs of the asset classes. In contrast to previous studies, we compare risk, return and asset allocation results based on real versus nominal returns, which makes the impact of the differing inflation-hedging abilities of the assets evident.

3. VAR model and data

3.1. VAR specification

The basic framework follows Campbell and Viceira (2005), who introduce a model for long-term buy-and-hold investors. Let z_{t+1} be a vector that includes log (continuously compounded) asset returns and additional state variables that predict returns. Assume that a VAR(1) model captures the dynamic relationships between asset returns and the additional state variables:

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[2] Hoevenaars et al. (2008), Porras Prado and Verbeek (2008) and Amenc et al. (2009b) extend the long-term asset allocation analysis based on VAR estimates to an asset-liability context, modelling the dynamics of liabilities of institutional investors.

[3] Sources: The IPD Index Guide, Edition 5, and EPRA Monthly Statistical Bulletin, December 2008.

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1}. \tag{1}$$

In the specification of this study, the nominal return on cash ($n_{0,t+1}$), and the excess returns on real estate, stocks, and long-term bonds (stacked in the (3x1) vector $\boldsymbol{x}_{t+1} = \boldsymbol{n}_{t+1} - n_{0,t+1}\boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is a vector of ones) are elements of $\boldsymbol{z}_{t+1}$. In addition, $\boldsymbol{z}_{t+1}$ contains the realization of inflation i_{t+1}, and three other state variables stacked in the (3x1) vector $\boldsymbol{s}_{t+1}$ (the cap rate, the dividend yield and the yield spread). Thus,

$$\boldsymbol{z}_{t+1} = \begin{bmatrix} n_{0,t+1} \\ \boldsymbol{x}_{t+1} \\ i_{t+1} \\ \boldsymbol{s}_{t+1} \end{bmatrix} \tag{2}$$

is of order (8x1). $\boldsymbol{\Phi}_0$ is a (8x1) vector of constants and $\boldsymbol{\Phi}_1$ is a (8x8) coefficient-matrix. The shocks are stacked in the (8x1) vector $\boldsymbol{v}_{t+1}$, and are assumed to be IID normal with zero means and covariance-matrix $\boldsymbol{\Sigma}_v$, which is of order (8x8):

$$\boldsymbol{v}_{t+1} \sim IIDN(\boldsymbol{0}, \boldsymbol{\Sigma}_v) \text{ with } \boldsymbol{\Sigma}_v = \begin{pmatrix} \sigma_0^2 & \boldsymbol{\sigma}_{0x}' & \sigma_{0i} & \boldsymbol{\sigma}_{0s}' \\ \boldsymbol{\sigma}_{0x} & \boldsymbol{\Sigma}_{xx} & \boldsymbol{\sigma}_{ix} & \boldsymbol{\Sigma}_{sx} \\ \sigma_{0i} & \boldsymbol{\sigma}_{ix}' & \sigma_i^2 & \boldsymbol{\sigma}_{is}' \\ \boldsymbol{\sigma}_{0s} & \boldsymbol{\Sigma}_{sx}' & \boldsymbol{\sigma}_{is} & \boldsymbol{\Sigma}_{ss} \end{pmatrix}. \tag{3}$$

The main diagonal of $\boldsymbol{\Sigma}_v$ consists of the variance of nominal cash return shocks, σ_0^2, the covariance-matrix of excess return shocks, $\boldsymbol{\Sigma}_{xx}$, the variance of inflation shocks, σ_i^2, and the covariance-matrix of the residuals of the state variables, $\boldsymbol{\Sigma}_{ss}$. The off-diagonal elements are the vector of covariances between shocks to the nominal return on cash and shocks to the excess returns on real estate, stocks and bonds, $\boldsymbol{\sigma}_{0x}$, the covariance of shocks to the nominal cash return with inflation shocks, σ_{0i}, the vector of covariances between shocks to the excess returns on real estate, stocks and bonds with inflation shocks, $\boldsymbol{\sigma}_{ix}$, the vector of covariances between shocks to the nominal cash return and shocks to the state variables, $\boldsymbol{\sigma}_{0s}$, the covariance matrix of shocks to the excess returns and shocks to the state variables, $\mathbf{\Sigma}_{sx}$, and the vector of covariances between inflation shocks and shocks to the state variables, $\boldsymbol{\sigma}_{is}$.

3.2. Data

The results are based on an annual dataset from 1957 to 2008 (52 observations) for the UK market; the Appendix provides details on the data used. As noted above, cash (T-bills), real estate, stocks and long-term bonds are the assets available to the investor. The bond index represents a security with constant maturity of 20 years. The implicit strategy assumed here is to sell a bond at the end of each year and buy a new bond to keep the bond maturity constant, an assumption which is common for bond indexes. As in Campbell and Viceira (2005), the log of the dividend yield of the stock market and the log yield spread, i.e., the difference between the log yield of a long-term bond and the log yield of T-bills are incorporated as state variables that have been shown to predict asset returns. We also include the (log of the) cap rate as a state variable that has been shown to predict direct real estate returns (Fu and Ng 2001, Ghysels et al. 2007, Plazzi et al. 2010).

Appraisal-based capital and income real estate returns used to calculate the annual real estate total return and the cap rate series have been obtained from two sources. The returns from 1971 to 2008 are based on IPD's long-term index. Initially, the index covered a portfolio of 651 properties, increasing to 11,328 properties by 1981 (Newell and Webb 1994). Returns from 1956 to 1970 are from Scott (1996).[4] These returns are based on valuations of properties in portfolios of two large financial institutions covering more than 1,000 properties throughout this period (Scott and Judge 2000).[5] Key et al. (1999) find that the Scott return series used here as well as the IPD 1971 to 1980 return series are fairly reliable in terms of coverage.

Real estate returns are unsmoothed using the approach of Barkham and Geltner (1994). This unsmoothing approach is based on modeling optimal behavior of property appraisers as

[4] Note that due to the unsmoothing procedure for real estate returns, one additional observation is needed.

[5] For comparison, the widely-used NCREIF Property Index (NPI) was based on 233 properties at the index inception; see "Frequently asked questions about NCREIF and the NCREIF Property Index (NPI)" on the NCREIF website (www.ncreif.org).

introduced by Geltner (1993). Appraisal-based log real capital returns g_t^* are unsmoothed using the formula

$$g_t = \frac{g_t^* - (1-a)g_{t-1}^*}{a}, \tag{4}$$

where g_t is the true log real capital return (or growth) and a is the smoothing parameter. We use the value 0.625 for unsmoothing annual returns as favored by Barkham and Geltner (1994). Total real estate returns and cap rates are constructed from the unsmoothed log real capital return and income return series; see the Appendix.

Table 1 provides an overview of the sample statistics of the variables used in the VAR model. Mean log returns of the assets are adjusted by one half of the variance to reflect log mean returns. Nominal cash returns are very persistent. Stocks have the highest mean return but also the highest volatility. Bonds have a mean excess return with regard to cash of only 1% p.a., but bond returns are quite volatile so that the Sharpe ratio is low. Real estate lies in between stocks and bonds with regard to volatility, mean return and Sharpe ratio. The unsmoothed real estate returns do not show notable autocorrelation. The state variables exhibit high persistency, especially the inflation rate. The inflation rate has a high mean and a high volatility. The cap rate has a higher mean and a lower volatility than the dividend yield of the stock market.

[Table 1 about here]

Figure 1 shows the logarithm of the (nominal) total return index values of the four asset classes and the development of the cost of living index. Real estate, bond and stock markets collapsed during the oil crisis of 1973-74. After that, the stock market was characterized by a long upswing until the turn of the century. The real estate market experienced a significant downturn in the early 1990s and again – like the stock market – in recent times. In general, bonds performed poorly. It took until the mid-1980s for bonds to have a higher index value than the consumer price index, and a decade later the bond index value exceeded that of cash investments. The cash index reflects the persistent behavior of cash returns.

[Figure 1 about here]

3.3. VAR estimates

The results of the VAR(1), estimated by OLS, are given in Table 2. Panel A contains the coefficients. In square brackets are *p*-values. Panel B contains the standard deviations (diagonal) and correlations (off-diagonals) of the VAR residuals.

[Table 2 about here]

The *p*-values of the *F*-test of joint significance indicate that the nominal return on cash and the excess returns on the other assets are indeed predictable. Especially nominal cash returns have a very high degree of predictability. The lagged yield spread is the most significant predictor of excess real estate returns. The yield spread tracks the business cycle (Fama and French 1989), so the relationship of real estate returns with the lagged yield spread points toward the close relationship with changes in GDP (Case et al. 1999, Quan and Titman 1999). Confirming previous studies, real estate returns can also be predicted by the cap rate.[6] The most significant predictor of stock returns is the dividend yield. The lagged yield spread is positively related to bond returns, albeit not significantly. Somewhat surprisingly, the cap rate is a significant predictor of excess bond returns. All state variables are highly significantly related to their own lag.

Turning to the correlations of the residuals, we see that excess stock and real estate return residuals are almost perfectly negatively correlated with shocks to the respective market yield (dividend yield and cap rate respectively). Unexpected nominal cash returns and unexpected inflation are positively correlated, while shocks to the excess return on bonds and inflation shocks are negatively correlated. Shocks to excess returns on real estate and stocks have a correlation of close to zero with unexpected inflation. However, even if return shocks are negatively correlated with inflations shocks, the asset may be a good long-term hedge against inflation.

[6] Gyourko and Keim (1992) as well as Barkham and Geltner (1995), among others, show that returns on direct real estate are positively related to lagged returns on property shares. It should be noted that returns on real estate stocks and general stocks are highly correlated, and general stocks are included in the VAR. Nevertheless, we recalculated the results in this paper with the excess return on property shares (UK Datastream real estate total return index) as an additional state variable for the period 1965 (the inception of the property share index) to 2008. The main results are similar to those reported in this paper. To make use of the additional observations and to avoid proliferation of the VAR parameters, the eight-variable VAR is used.

4. Horizon effects in risk and return for nominal and real returns

4.1. The term structure of risk

The risk statistics are based on the covariance matrix of the VAR residuals. Hence, we calculate conditional risk statistics, i.e., taking return predictability into account. The conditional multi-period covariance matrix of the vector $\boldsymbol{z}_{t+1}$, scaled by the investment horizon k, can be calculated as follows (see, e.g., Campbell and Viceira 2004):

$$\begin{aligned}\frac{1}{k}Var_t(\boldsymbol{z}_{t+1}+...+\boldsymbol{z}_{t+k}) = \frac{1}{k}[&\boldsymbol{\Sigma}_v+(\mathbf{I}+\boldsymbol{\Phi}_1)\boldsymbol{\Sigma}_v(\mathbf{I}+\boldsymbol{\Phi}_1)' \\ &+(\mathbf{I}+\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_1^2)\boldsymbol{\Sigma}_v(\mathbf{I}+\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_1^2)'+... \\ &+(\mathbf{I}+\boldsymbol{\Phi}_1+...+\boldsymbol{\Phi}_1^{k-1})\boldsymbol{\Sigma}_v(\mathbf{I}+\boldsymbol{\Phi}_1+...+\boldsymbol{\Phi}_1^{k-1})'],\end{aligned} \tag{5}$$

where $\mathbf{I}$ is the (8x8) identity matrix. The conditional covariance matrix of nominal returns and inflation can be calculated from the conditional multi-period covariance matrix of $\boldsymbol{z}_{t+1}$, using the selector matrix

$$\mathbf{M}_n = \begin{bmatrix} 1 & \mathbf{0}_{1x3} & 0 & \mathbf{0}_{1x3} \\ \boldsymbol{\iota}_{3x1} & \mathbf{I}_{3x3} & \mathbf{0}_{3x1} & \mathbf{0}_{3x3} \\ 0 & \mathbf{0}_{1x3} & 1 & \mathbf{0}_{1x3} \end{bmatrix}. \tag{6}$$

Nominal return statistics can be calculated because the vector $\boldsymbol{z}_{t+1}$ includes the nominal cash return and excess returns such that the nominal return statistics of stocks, bonds and direct real estate can be calculated by adding the nominal cash return and the excess return of the respective asset:

$$\frac{1}{k}Var_t\begin{bmatrix} n_{0,t+k}^{(k)} \\ \boldsymbol{n}_{t+k}^{(k)} \\ i_{t+k}^{(k)} \end{bmatrix} = \frac{1}{k}\mathbf{M}_n Var_t(\boldsymbol{z}_{t+1}+...+\boldsymbol{z}_{t+k})\mathbf{M}_n^{'} . \tag{7}$$

Similarly, real return statistics can be calculated using the selector matrix

$$\mathbf{M}_r = \begin{bmatrix} \mathbf{I}_{4x4} & -\mathbf{1}_{4x1} \\ \mathbf{0}_{1x4} & 1 \end{bmatrix}, \tag{8}$$

such that the *k*-period conditional covariance matrix of real returns and inflation, per period, is:

$$\frac{1}{k} Var_t \begin{bmatrix} r_{0,t+k}^{(k)} \\ \boldsymbol{r}_{t+k}^{(k)} \\ i_{t+k}^{(k)} \end{bmatrix} = \frac{1}{k} \mathbf{M}_r Var_t \begin{bmatrix} n_{0,t+k}^{(k)} \\ \boldsymbol{n}_{t+k}^{(k)} \\ i_{t+k}^{(k)} \end{bmatrix} \mathbf{M}_\mathrm{r}', \tag{9}$$

where $r_{0,t+k}^{(k)}$ is the *k*-period real return on cash (the benchmark asset) and $\boldsymbol{r}_{t+k}^{(k)}$ is the vector of *k*-period real returns on real estate, stocks and bonds.

The annualized standard deviations for nominal and real returns of the four asset classes, depending on the investment horizon, are shown in Figure 2. Due to return persistency, the periodic long-term return volatility of real cash returns is much higher than the short-term volatility. The mean aversion effect is even more pronounced for nominal returns. For long investment horizons, the volatility of nominal returns is notably higher than the volatility of real returns. Real stock returns are mean reverting. Nominal stock returns are mean reverting over short investment horizons, too, but then the term structure is increasing to such an extent that the periodic long-term volatility of nominal returns in higher than the long-term volatility of real returns. Nominal bond returns are less volatile than real returns for all investment horizons, but the 20-year volatilities are quite similar. Recall that we use a constant maturity bond index. While a 20-year (zero-) bond held to maturity is riskless in nominal terms, this is not true for a 20-year constant maturity bond index. Qualitatively, the results for real cash, stock and bond returns are similar to the US results reported in Campbell and Viceira (2005), except that they find that bond returns are slightly mean reverting. Nominal and real returns on real estate are mean reverting. For medium and long-horizons, however, the annualized volatility of nominal returns is higher than the volatility of real returns. The mean reversion effect in real stock and real estate returns can be explained by the positive relation of excess returns on the lagged market yield (dividend yield and cap rate respectively) and the high negative correlation of return shocks and market yield shocks. If (property or stock) prices are decreasing, this is bad news for an investor. On the other hand, the good news is that a low realized return on stocks (real estate) is usually accompanied by a positive shock to the dividend yield (cap rate), and a high dividend yield (cap rate) predicts high returns for the future.

[Figure 2 about here]

Figure 3 shows horizon-dependent asset correlations for both nominal and real returns, as implied by the VAR estimates. The correlation between real stock and bond returns at medium investment horizons is higher, but the long-term correlation is lower than the short-term correlation. This is similar to the Campbell and Viceira (2005) estimates. The correlation between real stock and real estate returns is slightly lower in the long run than in the short run. The correlation between the real returns on real estate and cash is strongly increasing with the investment horizon. Real bond and stock returns are highly correlated with cash returns in the long-term, too. In general, the long-term correlations of real asset returns are less dispersed than the short-term correlations, which is intuitively appealing. With the exception of the correlation between cash and bonds, the long-term correlations of nominal returns are higher than the long-term correlations of real returns, pointing towards inflation as a common driver of long-term nominal asset returns. In contrast, the short-term correlation of nominal cash and stock returns and in particular the short-term correlation of nominal cash and bond returns is notably lower than the respective correlation of real returns. Hence, inflation affects nominal cash and stock returns, and nominal cash and bonds returns differently in the short run.

[Figure 3 about here]

4.2. Inflation hedging

To gain deeper insights into the differences between the term structures of return volatility for real and nominal returns, we derive formulas for the variance of nominal and real returns based on the approximation for the k-period portfolio return introduced by Campbell and Viceira (2002) and used in Campbell and Viceira (2004, 2005). Accounting for transaction costs regarding real estate and stock and bond investments, stacked in the (3x1) vector $\boldsymbol{c}$, the approximation to the nominal k-period portfolio return is:

$$n_{p,t+k}^{(k)} = n_{0,t+k}^{(k)} + \boldsymbol{\alpha}'(k)(\boldsymbol{x}_{t+k}^{(k)} - \boldsymbol{c}) + \tfrac{1}{2}\boldsymbol{\alpha}'(k)[\boldsymbol{\sigma}_x^2(k) - \boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\alpha}(k)], \qquad (10a)$$

where $\boldsymbol{\alpha}(k)$ is the (3x1) vector containing the asset weights, except for the weight on cash, with regard to a *k*-period investment, and $\boldsymbol{\sigma}_x^2(k) = diag[\boldsymbol{\Sigma}_{xx}(k)]$. Subtracting the *k*-period inflation rate $i_{t+k}^{(k)}$ yields the real portfolio return:

$$r_{p,t+k}^{(k)} = n_{0,t+k}^{(k)} + \boldsymbol{\alpha}'(k)(\boldsymbol{x}_{t+k}^{(k)} - \boldsymbol{c}) + \tfrac{1}{2}\boldsymbol{\alpha}'(k)[\boldsymbol{\sigma}_x^2(k) - \boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\alpha}(k)] - i_{t+k}^{(k)}, \qquad (10b)$$

From (10) one can calculate the conditional *k*-period variance of the portfolio return as:

$$Var_t(n_{p,t+k}^{(k)}) = \boldsymbol{\alpha}'(k)\boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\alpha}(k) + \sigma_0^2(k) + 2\boldsymbol{\alpha}'(k)\boldsymbol{\sigma}_{0x}(k) \qquad (11a)$$

$$Var_t(r_{p,t+k}^{(k)}) = \boldsymbol{\alpha}'(k)\boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\alpha}(k) + \sigma_0^2(k) + 2\boldsymbol{\alpha}'(k)\boldsymbol{\sigma}_{0x}(k) + \sigma_i^2(k) - 2\sigma_{0i}(k) - 2\boldsymbol{\alpha}'(k)\boldsymbol{\sigma}_{ix}(k). \qquad (11b)$$

Assuming a 100% investment in the respective asset, equations (11a) and (11b) are the formulas for the variance of asset returns. The variance of the nominal return on an asset differs from the variance of the real return on the asset by the last three terms in (11b). The first of the three terms says that for all assets the real return volatility is higher than the nominal return volatility due to the variance of inflation shocks. The annualized *k*-period standard deviation of inflation shocks is shown in Figure 4.

[Figure 4 about here]

We see that due to the persistence of inflation, the periodic long-term volatility of inflation is much larger than the short-term volatility. Ceteris paribus, this significantly increases the long-term volatility of real returns. There are two additional terms with regard to the differences between the volatility of nominal and real returns, though. When the conditional covariance between nominal cash returns and inflation, σ_{0i}, is positive, this decreases the volatility of real cash returns. For the analysis of the other assets it is helpful to note that

$$-2\sigma_{0i}(k) - 2\boldsymbol{\alpha}'(k)\boldsymbol{\sigma}_{ix}(k) = -2\boldsymbol{\alpha}'(k)\boldsymbol{\sigma}_{in}(k) - 2(1 - \boldsymbol{\alpha}'(k)\boldsymbol{\iota})\sigma_{0i}(k), \qquad (12)$$

where $\boldsymbol{\sigma}_{in}$ is the vector of covariances between inflation shocks and shocks to the nominal returns on real estate, stocks and bonds. The last term on the right hand side of (12) is zero for a 100% investment in real estate, stocks or bonds. Therefore, we see again that the conditional covariance of the nominal asset return with inflation is crucial for the difference between the variance of real versus the variance of nominal returns. Recall that the horizon-dependences of nominal return volatilities and of inflation volatility are shown in Figures 2 and 4 respectively. What we are missing to analyze the covariances are the horizon-dependent correlations of nominal asset returns with inflation, and these are shown in Figure 5. Cash is clearly the best inflation-hedging asset at short and medium horizons. Shocks to nominal cash returns are relatively highly correlated with inflation shocks and the correlation is increasing with the investment horizon. At the twenty-year horizon, real estate appears to hedge inflation as well as cash. Bonds are the weakest inflation-hedging asset in the short-term. In the long run, bonds and stocks have much better inflation hedging abilities than in the short run.

[Figure 5 about here]

There are theoretical arguments supporting this empirical evidence. Fama and French (1977) point out that a strategy of rolling over short-term bills should offer a good hedge against longer-term unexpected inflation because short-term bill rates can adjust to reassessments of expected inflation. In contrast to this strategy, the cash-flows of a (default risk-free) nominal long-term bond are fixed, so the nominal long-term return does not move with inflation. Standard bond indexes, such as the one used in this paper, are, however, representing a security with constant maturity. In terms of inflation hedging, this means that the return on these bond indexes benefits from the reassessments of expected inflation that are incorporated into the bond yield, so that the ability of constant maturity bond returns to hedge unexpected inflation improves with the investment horizon. Campbell and Vuolteenaho (2004) suggest that the finding of stocks is a perverse inflation-hedge in the short run, but a good inflation-hedge in the long run can be explained by money illusion. They find empirical support for the Modigliani and Cohn (1979) hypothesis, who conclude that stock market investors suffer from a specific form of money illusion, disregarding the effect of changing inflation on cash-flow growth. When inflation rises unexpectedly, investors increase discount rates but ignore the impact of expected inflation on expected cash-flows, leading to an undervalued stock market, and vice versa. Because the misevaluation should eventually diminish, stocks are a good inflation-hedge in the long run. Direct real estate has both stock and bond characteristics. Bond characteristics are due to the contractual rent representing a fixed-claim against the tenant. However, rents are routinely adjusted to market level through renting vacant space or arrangements in the lease contract. For example, in the UK commercial real estate market, contractual rents are usually reviewed every five years; they are adjusted to market-rent level, when this level is above the contractual rent, otherwise the contractual rent remains unchanged. Thus, when general price and rent indexes are closely related, direct real estate should be a good long-term inflation hedge.

These inflation-hedging patterns help to reinterpret the findings shown in Figure 2. With regard to cash, we see that in the short-term the effect of the addition of the inflation variance dominates the covariance effect such that the volatility of real returns is slightly higher than the volatility of nominal returns. In the long-run, however, the increasing correlation of nominal cash returns with inflation makes real cash returns less volatile than nominal cash returns. For short horizons, the correlation between nominal stock returns and inflation is low, and therefore, real stock returns are more volatile than nominal stock returns. The correlation of nominal returns with inflation, however, increases with the investment horizon, so that the long-term volatility of nominal returns is higher than the volatility of real returns. For bond returns, the effect of the addition of the inflation volatility dominates the covariance effect for all horizons. But as the correlation between nominal bond returns and inflation increases with the investment horizon, the long-term standard deviations of real and nominal bond returns are quite similar. For real estate, the correlation between nominal returns and inflation is strongly increasing with the investment horizon, so that the volatility of real returns is notably smaller than the volatility of nominal returns in the long run.

4.3. The term structure of expected returns

From (10a) and (11a) one can calculate the k-period log expected nominal portfolio return as:

$$E(n_{p,t+k}^{(k)}) + \tfrac{1}{2} Var_t(n_{p,t+k}^{(k)}) = E(n_{0,t+k}^{(k)}) + \tfrac{1}{2}\sigma_0^2(k) + \boldsymbol{\alpha}'[E(\boldsymbol{x}_{t+k}^{(k)} - \boldsymbol{c}) + \tfrac{1}{2}\boldsymbol{\sigma}_x^2(k) + \boldsymbol{\sigma}_{0x}(k)]. \tag{13a}$$

This equation shows how to calculate the (approximation of the) cumulative log expected nominal portfolio return or, assuming a 100% investment in the respective asset, the log expected nominal return of any single asset class. Note that the expected log return has to be adjusted by one half of the return variance to obtain the log expected return relevant for portfolio optimization (a Jensen's inequality adjustment); see Campbell and Viceira (2004). This adjustment is horizon-dependent. There are no horizon effects in expected log returns because we assume that they take the values of their sample counterparts. Thus, for the k-period expected log nominal cash return it holds that $E(n_{0,t+k}^{(k)}) = k\bar{n}_0$, where $\bar{n}_0$ denotes the sample average of log nominal cash returns. Similarly, we assume for the vector of log excess returns: $E(\boldsymbol{x}_{t+k}^{(k)}) = k\bar{\boldsymbol{x}}$. Even if there were no horizon effects in expected log returns, there would be horizon effects in log expected returns because conditional variances and covariances will not increase in proportion to the investment horizon unless returns are unpredictable. In the remainder of this paper, the log expected return is termed “expected return” for short.

Additional horizon effects in expected returns are due to the consideration of proportional transaction costs. With regard to stocks and bonds, transaction costs encompass brokerage commissions and bid-ask spreads. Round-trip transaction costs for stocks are assumed to be 1.0%, as in Balduzzi and Lynch (1999) and Collet et al. (2003). Bid-ask spreads of government bonds are typically tiny (Fleming 2003, Gwilym et al. 2002); total round-trip transaction costs for bonds, including brokerage commissions, are assumed to be 0.1%. Transaction costs for buying and selling real estate encompass professional fees and the transfer tax. According to Collet et al. (2003), round-trip transaction costs for UK real estate are 7 to 8%. Marcato and Key (2005) assume round-trip transaction costs of 7.5%. These costs cover the transfer tax (“stamp duty”) of 4.0% (to be paid when buying), 1.5% for legal, agents’ and other advisory fees for both purchases and sales, plus 0.5% internal investor's costs. We exclude the internal costs and hence assume total costs of 7.0%, which appears to be reasonable as Marcato and Key suggest that 7.5% may be a bit on the high side. The costs are divided into 5.5% buying costs and 1.5% selling costs. Round-trip transaction costs for stocks and bonds are divided by one half to obtain the costs for buying and selling separately. The assumed round-trip transaction costs enter the vector $\boldsymbol{c}$ in continuously compounded form, and they are obtained by adding the continuously compounded buying and selling costs, so that $\boldsymbol{c}' = [6.84\% \quad 1.00\% \quad 0.10\%]$. For example, the round-trip costs for real estate are $\ln(1.055) + \ln(1.015) \approx 6.84\%$.

The k-period expected real portfolio return can be calculated from (10b) and (11b) as:

$$E(r_{p,t+k}^{(k)})+\tfrac{1}{2}Var_t(r_{p,t+k}^{(k)})=E(n_{0,t+k}^{(k)})+\tfrac{1}{2}\sigma_0^2(k)-E(i_{t+k}^{(k)})+\tfrac{1}{2}\sigma_i^2(k)$$
$$+\boldsymbol{\alpha}'[E(\boldsymbol{x}_{t+k}^{(k)}-\boldsymbol{c})+\tfrac{1}{2}\boldsymbol{\sigma}_x^2(k)+\boldsymbol{\sigma}_{0x}(k)]-\sigma_{0i}(k)-\boldsymbol{\alpha}'\boldsymbol{\sigma}_{ix}(k), \tag{13b}$$

where $E(i_{t+k}^{(k)})=k\bar{i}$, the k-period expected log inflation and $\tfrac{1}{2}\sigma_i^2(k)$, one-half of the cumulative variance of inflation shocks, are common differences for the distinction between nominal expected returns and real expected returns for every asset. In addition, the conditional covariances between asset returns and inflation ($\sigma_{0i}(k)$ and $\boldsymbol{\sigma}_{ix}(k)$ respectively) play a role. The results of the comparison between the term structures of annualized expected real and nominal returns after transaction costs for cash, real estate, stocks, and bonds are shown in Figure 6.

[Figure 6 about here]

The difference between the expected real and nominal returns is a nearly parallel shift caused by the expected inflation. Due to transaction costs, there are major changes in the annualized expected real estate return, which increases strongly with the investment horizon, whereas the periodic expected returns on the other assets are roughly constant.

5. Horizon-dependent portfolio optimizations for nominal and real returns

5.1. Mean-variance optimization

Campbell and Viceira (2002, 2004) provide the formula for the solution to the mean-variance problem. Augmented by transactions, this is:

$$\boldsymbol{\alpha}(k)=\tfrac{1}{\gamma}\boldsymbol{\Sigma}_{xx}^{-1}(k)[E(\boldsymbol{x}_{t+k}^{(k)}-\boldsymbol{c})+\tfrac{1}{2}\boldsymbol{\sigma}_x^2(k)]+(1-\tfrac{1}{\gamma})[-\boldsymbol{\Sigma}_{xx}^{-1}(k)\boldsymbol{\sigma}_{0x}(k)], \tag{14}$$

where γ is the coefficient of relative risk aversion. $\boldsymbol{\alpha}(k)$ is a combination of two portfolios; the second portfolio is the global minimum variance portfolio:

$$\min\,[\text{w.r.t. }\boldsymbol{\alpha}(k)]\ \frac{1}{2}Var_t(n_{p,t+k}^{(k)})=-\boldsymbol{\Sigma}_{xx}^{-1}(k)\boldsymbol{\sigma}_{0x}(k). \tag{15}$$

Formula (14) applies directly to the mean-variance problem for nominal returns. The solution to the mean-variance problem for real returns differs from (14) only by the definition of the global minimum variance portfolio, which for real returns is:

$$\min\,[\text{w.r.t. } \boldsymbol{\alpha}(k)]\ \frac{1}{2} Var_t(r_{p,t+k}^{(k)}), \tag{16}$$

where $Var_t(r_{p,t+k}^{(k)})$ is defined in (11b).

We analyze two portfolios. One portfolio is the global minimum variance portfolio. As in Campbell and Viceira (2005), we exclude cash as an available asset for the second portfolio, which represents a less risk-adverse investor than the global minimum-variance investor. Campbell and Viceira calculate a "tangency-portfolio" assuming that there would be a riskless asset. This is not suitable for our analysis because we would have to assume that both real and nominal cash returns would be riskless (at any horizon) and hence there would be no inflation risk. Therefore, we calculate optimal horizon-dependent asset weights for a portfolio consisting of bonds, stocks and real estate for a specific coefficient of relative risk aversion; we choose $\gamma = 5.$ The formulas still apply to this restricted investment universe, except that not cash is the benchmark asset, but bonds, i.e., for the second portfolio $n_{0,t+1}$ is not the nominal return on cash, but on bonds, and $\boldsymbol{x}_{t+1}$ refers to the excess returns on real estate and stocks with regard to the return on bonds. The necessary statistics can be calculated by applying appropriate selection matrixes to (7) and (9). We rule out short-selling of direct real estate but do no impose short-selling restrictions for the other assets.

5.2. Results

Figure 7 shows two Panels with optimal portfolio allocations for investment horizons of up to twenty years. Panel A plots the composition of the global minimum variance (GMV) portfolio for optimizations based on real and nominal returns. At the one-year horizon, the differences between the allocations based on real returns versus the allocations based on nominal returns are small. A very risk-averse investor holds most of his money in cash because it is the least risky investment in nominal as well as in real terms over all investment horizons. However, as the annualized volatility strongly increases with the investment horizon, the weight assigned to cash decreases. Since the increase in the return volatility is stronger in nominal terms, this decrease is stronger for the optimization based on nominal returns. The weight assigned to real estate is increasing with the investment horizon. Again, the differences between the term structures of return volatility for nominal and real returns are crucial for the extent of the horizon effect. For the optimization based on nominal returns, the allocation to real estate increases to 10% at intermediate and up to 25.5% at long horizon. For real returns, the mean reversion effect is stronger and hence the weight assigned to real estate is much higher than the allocation for nominal returns at medium and long horizons. Bonds are more attractive in nominal than in real terms. Based on the optimizations for nominal returns, the weight increases from 3.6% at the one-year horizon to 32.8% at the twenty-year horizon, whereas the weight is negative for all investment horizons when real returns are considered. Because of the hump-shaped risk structure of nominal stock returns (high short- and long-term volatility – less risky in the medium term), stocks get a small positive weight at medium investment horizons and get a slightly negative weight at long investment horizons. For real returns, the allocation to stocks is negative for all investment horizons.

[Figure 7 about here]

Panel B reports the portfolio allocation comparison for an investor with moderate risk ($\gamma = 5$). As noted above, we only consider stocks, bonds and real estate. In addition to risk statistic, the term structures of expected returns are relevant for this portfolio. Recall that the differences with regard to nominal versus real returns are roughly parallel shifts. Hence, when comparing the results for nominal and real returns, the changing risk statistics are again crucial for the interpretation. We see once more that the differences in optimal portfolio weights are small at short horizons, since short-term return volatilities are similar for real and nominal returns. As in the GMV portfolio, real estate is much more attractive in the long run. Due to the short-selling restriction the allocation is zero at the one-year horizon. The weight increases to 50.6% for real returns and to 25.6% for nominal returns at the twenty-year horizon. For the optimization based on real returns, the allocation to stocks rises smoothly from 24.6% at the one-year to 53.8% at the twenty-year horizon due to the strong mean reversion effect of real stock returns. For the optimization based on nominal returns, the allocation to stocks is more variable and shows a hump-shaped structure with high allocations at intermediate horizons (up to 68.3%) and only 25% (40%) at short (long) horizons. Due to the low expected return on real estate and the high volatility of stock returns, bonds are the asset with the highest allocation at short horizons. The weight assigned to bonds is strongly decreasing with the investment horizon for the allocation based on real returns, since the term structure of the periodic return volatility is roughly flat, whereas stocks and real estate are getting more attractive with the investment horizon. In nominal terms, however, the weight assigned to bonds is increasing over longer investment horizons because stocks are getting very unattractive due to the increase in the periodic return volatility, which is stronger than the mean aversion of nominal bond returns over long horizons.

In summary, good inflation hedging assets classes increase their weights when the optimization is based on real rather than nominal returns. This is true for cash over all and for real estate over medium and long investment horizons. Stocks become less attractive for medium horizons, but due to the good inflation hedging abilities over long horizons, the long-term allocation to stocks is higher when the optimization is based on real instead of nominal returns. Bonds become less attractive for all investment horizons.

6. Robustness of the results with regard to the smoothing parameter

We recalculate main results for investment horizons of one, five, ten and twenty years for alternative parameter values used to unsmooth the appraisal-based real estate returns. Two alternative parameter values are considered, which Barkham and Geltner (1994) consider as reasonable lower and upper bounds: $a = 0.50$ and $a = 0.75$. The results are presented in Table 3. For comparison, the results obtained from the assumption made so far ($a = 0.625$) are also reported. We ignore (small) changes in the mean return that result from unsmoothing returns with different parameters. The results for cash, bonds, and stocks are largely unaffected by the choice of the smoothing parameter; the results presented therefore focus on real estate.

[Table 3 about here]

The choice of the smoothing parameter has a large impact on the conditional standard deviation of the return on real estate at the one-year horizon in both nominal and real terms. When it is assumed that the original returns suffer from a lot of smoothing ($a = 0.50$), the one-year volatility is about 17.5%. In contrast, when the original returns are assumed to exhibit relatively little smoothing ($a = 0.75$), the one-year volatility is less than 12%. However, the longer the investment horizon, the smaller this difference is. At the twenty-year horizon, there is almost no difference. Due to the Jensen's inequality adjustment, expected returns are higher for $a = 0.5$; again, the longer the investment horizon, the smaller the differences are. Correlations of nominal returns on real estate with inflation are quite similar under the different smoothing parameters. In general, the allocation to real estate is lower when the original real estate returns are assumed to be more smoothed $(a = 0.5)$ since this yields more volatile unsmoothed returns, but the differences are not very large. Overall, the results appear to be fairly robust to changes in the smoothing parameter.

7. Conclusion

Focusing on the role of the investment horizon, we analyze the inflation-hedging abilities of stocks, bonds, cash and direct commercial real estate investments, and the implications of the inflation-hedge results for portfolio choice. Based on vector autoregressions for the UK market we find that the inflation-hedging abilities of all assets analyzed improve with the investment horizon. Cash is clearly the best inflation hedge at short and medium horizons. For long horizons, real estate hedges unexpected inflation as well as cash. This has implications for the difference between the return volatility of real returns versus the return volatility of nominal returns. The long-term volatility of real returns on real estate is notably lower than the long-term volatility of nominal returns. This is also true for cash returns. In contrast, bonds are less attractive for an investor concerned about inflation. The same is found for stocks at medium horizons, but at long horizons the volatility of real stock returns is lower than the volatility of nominal returns. Portfolio optimizations based on real returns yield higher allocations to cash and real estate than optimizations based on nominal returns. Bonds tend to be less attractive for an investor taking into account inflation. Switching from nominal to real returns, the allocation to stocks is decreasing at medium investment horizon, but increasing at long horizons. The differences between the asset allocation results can be substantial. This means that the optimal asset allocation for investors concerned about inflation (private investors and certain institutional investors) can be quite different from the optimal asset allocation for (institutional) investors with liabilities that are fixed in nominal terms.

References

Akerlof, G. A., Shiller, R. J., 2009. Animal Spirits. Princeton University Press: Princeton and Oxford.

Amenc, N., Martellini, L., Milhau, V., Ziemann, V., 2009a. Asset-liability management in private wealth management. Journal of Portfolio Management 36(1), 100-120.

Amenc, N., Martellini, L., Ziemann, V., 2009b. Inflation-hedging properties of real assets and implications for asset–liability management decisions. Journal of Portfolio Management 35(4), 94-110.

Balduzzi, P., Lynch, A. W., 1999. Transaction costs and predictability: Some utility cost calculations. Journal of Financial Economics 52, 47-78.

Barkham, R., Geltner, D., 1994. Unsmoothing british valuation-based returns without assuming an efficient market. Journal of Property Research 11, 81-95.

Barkham, R., Geltner, D., 1995. Price discovery in American and British property markets. Real Estate Economics 23, 21-44.

Bodie, Z., 1976. Common stocks as a hedge against inflation. Journal of Finance 31, 459-470.

Boudoukh, J., Richardson, M., 1993. Stock returns and inflation: A long-horizon perspective. American Economic Review 83, 1346-1355.

Brueggeman, W. B., Chen, A. H., Thibodeau, T. G., 1984. Real estate investment funds: Performance and portfolio considerations. AREUEA Journal 12, 333-354.

Campbell, J. Y., Viceira L. M., 2002. Strategic Asset Allocation. Oxford University Press: Oxford and New York.

Campbell, J. Y., Viceira L. M., 2004. Long-horizon mean-variance analysis: A user guide. Manuscript, Harvard University.

Campbell, J. Y., Viceira L. M., 2005. The term structure of the risk-return trade-off. Financial Analysts Journal 61(1), 34-44.

Campbell, J. Y., Vuolteenaho, T., 2004. Inflation illusion and stock prices. American Economic Review (Papers and Proceedings) 94, 19-23.

Case, B., Goetzmann, W., Rouwenhorst, K. G., 1999. Global Real Estate Markets: Cycles and Fundamentals. Working Paper, Yale International Center for Finance.

Collet, D., Lizieri, C., Ward, C., 2003. Timing and the holding periods of institutional real estate. Real Estate Economics 31, 205-222.

Fama, E. F., Schwert, G. W., 1977. Asset returns and inflation. Journal of Financial Economics 5, 115-146.

Fama, E. F., French, K. R., 1989. Business conditions and expected returns on stocks and bonds. Journal of Financial Economics 25, 23-49.

Fisher, J. D., Young, M. S., 2000. Holding periods for institutional real estate in the NCREIF database. Real Estate Finance 17(3), 27-34.

Fleming, M. J., 2003. Measuring treasury market liquidity. Economic Policy Review 9(3), 83-108.

Fu, Y., Ng, L. K., 2001. Market efficiency and return statistics: Evidence from real estate and stock markets using a present-value approach. Real Estate Economics 29, 227-250.

Fugazza, C., Guidolin, M., Nicodano G., 2007. Investing for the long-run in european real estate. Journal of Real Estate Finance and Economics 34, 35-80.

Geltner, D. 1993. Estimating market values from appraised values without assuming an efficient market. Journal of Real Estate Research 8, 325-345.

Geltner, D., Miller, N. G., Clayton J., Eichholtz, P., 2007. Commercial real estate analysis and investment. 2nd edition. Thomson: Mason, Ohio.

Ghysels, E., Plazzi, A., Valkanov, R., 2007. Valuation in US commercial real estate. European Financial Management 13, 472-497.

Gultekin, N. B. 1983. Stock market returns and inflation: Evidence from other countries. Journal of Finance 38, 49-65.

Gwilym, O. A., Trevino, L., Thomas, S., 2002. Bid-ask spreads and the liquidity of international bonds. Journal of Fixed Income 12(2), 82-91.

Gyouko, J., Keim, D. B., 1992. What does the stock market tell us about real estate returns? AREUEA Journal 20, 457-485.

Gyourko, J., Linneman, P., 1988. Owner-occupied homes, income-producing properties, and REITs as inflation hedges: Empirical findings. Journal of Real Estate Finance and Economics 1, 347-372.

Hartzell, D., Hekman, J. S., Miles M. E., 1987. Real estate returns and inflation. AREUEA Journal 15, 617-637.

Hoesli, M., Lizieri, C., MacGregor, B., 2008. The inflation hedging characteristics of US and UK investments: A multi-factor error correction approach. Journal of Real Estate Finance and Economics 36, 183-206.

Hoevenaars, R., Molenaar, R., Schotman P., Steenkamp, T., 2008. Strategic asset allocation with liabilities: Beyond stocks and bonds. Journal of Economic Dynamics and Control 32, 2939-2970.

Jaffe, J. F., Mandelker, G., 1976. The “Fisher effect” for risky assets: An empirical investigation. Journal of Finance 31, 447-458.

Key, T., Zarkesh, F., Haq, N., 1999. The UK Property Cycle – a History from 1921 to 1997. RICS Research: London.

Limmack, R. J., Ward, C. W. R., 1988. Property returns and inflation. Land Development Studies 5, 47-55.

Liu, C. H., Hartzell, D. J., Hoesli, M. E., 1997. International evidence on real estate securities as an inflation hedge. Real Estate Economics 25, 193-221

MacKinnon, G. H., Al Zaman, A., 2009. Real estate for the long term: The effect of return predictability on long-horizon allocations. Real Estate Economics 37, 117-153.

Marcato, G., Key, T., 2005. Direct Investment in Real Estate – Momentum Profits and Their Robustness to Trading Costs. Journal of Portfolio Management 31(5), 55-69.

Markowitz, H., 1952. Portfolio Selection. Journal of Finance 7, 77-91.

Maurer, R., Sebastian, S. 2002. Inflation risk analysis of European real estate securities. Journal of Real Estate Research 24, 47-77.

Modigliani, F., Cohn, R. A., 1979. Inflation, rational valuation and the market. Financial Analysts Journal 35(2), 24-44.

Newell G., Webb, J. R., 1994. Institutional real estate performance benchmarks: An international perspective. Journal of Real Estate Literature 2, 215-223.

Plazzi, A., Torous, W., Valkanov, R., 2010. Expected returns and expected growth in rents of commercial real estate. Review of Financial Studies, forthcoming.

Porras Prado, M., Verbeek, M., 2008. Real estate in an ALM framework – The case of return predictability. Working Paper, RSM Erasmus University.

Quan, D. C., Titman, S., 1999. Do real estate prices and stock prices move together? An international analysis. Real Estate Economics 27, 183-207.

Rubens, J. H., Bond, M. T., Webb, J. R., 1989. The inflation-hedging effectiveness of real estate. Journal of Real Estate Research 4, 45-55.

Schätz, A., Sebastian, S., 2009. The links between property and the economy – Evidence from the British and German markets. Journal of Property Research 26, 171-191.

Schotman, P. C., Schweitzer M., 2000. Horizon sensitivity of the inflation hedge of stocks. Journal of Empirical Finance 7, 301-315.

Scott, P., 1996. The Property Masters – A History of the British Commercial Property Sector. Spon: London.

Scott, P., Judge, G., 2000. Cycles and steps in British commercial property values. Applied Economics 32, 1287-1297.

Appendix: Data

Table A1 contains information on the data.

Table A1: Data information

	Description	Source
Cash return	Change (%) of Barclays UK treasury bill index	Barclays Equity Guilt Study 2009
Cash yield	UK clearing banks base rate	Datastream
Bond yield	Yield of Barclays gilt index	Barclays Equity Guilt Study 2009
Stock return	Change (%) of Barclays equity index	Barclays Equity Guilt Study 2009
Bond return	Change (%) of Barclays gilt index	Barclays Equity Guilt Study 2009
Real estate return	Constructed as described in this Appendix	Scott (1996), IPD
Inflation	Change (%) of UK cost of living index	Barclays Equity Guilt Study 2009
Cap rate	Constructed as described in this Appendix	Scott (1996), IPD
Dividend yield	Income yield of Barclays equity index	Barclays Equity Guilt Study 2009

The real estate total return and cap rate series are calculated as follows: The unsmoothed log real capital returns (see section 3.2. for a description of the unsmoothing procedure) are converted to simple nominal capital returns (CRU_t). This series is used to construct an unsmoothed capital value index (UCV_t). The unsmoothed capital value index is calibrated such that the average of the capital values over time matches the corresponding average of the original index. A real estate income series (Inc_t) is obtained by multiplying the (original) income return (IR_t) with the (original) capital value index (CV_t): $Inc_t = IR_t \cdot CV_{t-1}$. New income returns are computed with regard to the unsmoothed capital value index: $IRU_t = Inc_t / UCV_{t-1}$. Total returns are obtained by adding the adjusted simple income and capital returns: $RER_t = CRU_t + IRU_t$. The cap rate series is calculated as $CR_t = Inc_t / UCV_t$.

Table 1: Sample statistics

This table shows statistics for the variables included in the VAR model for the annual dataset (1957 to 2008). Autocorrelation refers to the first-order autocorrelation.

	Mean	Standard deviation	Auto-correlation	Sharpe ratio
Nominal return on cash*	7.62%	3.14%	84.58%	-
Excess return on real estate*	3.21%	15.57%	5.37%	0.2060
Excess return on stocks*	6.76%	23.76%	-13.69%	0.2845
Excess return on bonds*	0.97%	11.50%	-13.00%	0.0845
Log inflation	5.96%	4.57%	80.80%	-
Log of cap rate	-2.8416	0.2230	64.05%	-
Log of dividend yield	-3.1613	0.3113	69.53%	-
Log yield spread	0.30%	1.80%	43.76%	-

*Mean log returns are adjusted by one half of the return variance to reflect log mean returns.

Table 2: VAR results

The results are based on annual data from 1957 to 2008. Panel A shows the VAR coefficients. The *p*-values are given in square brackets; *p*-values of 10% or below are highlighted. The rightmost column contains the R^2 values and the *p*-value of the *F*-test of joint significance in parentheses. Panel B shows results regarding the covariance matrix of residuals, where standard deviations are on the diagonal and correlations are on the off-diagonals.

Panel A: VAR coefficients

		Coefficents on lagged variables								
Variable	Constant	1	2	3	4	5	6	7	8	R^2 (p)
1 Log nominal cash return	0.0135	0.7926	0.0579	0.0084	-0.0665	0.083	-0.0104	0.0105	-0.27	86.92%
	(0.751)	**(0.000)**	**(0.004)**	(0.431)	**(0.002)**	(0.225)	(0.346)	(0.201)	**(0.051)**	(0.00%)
2 Log real estate excess return	0.8872	-0.4507	0.1419	-0.0117	0.2316	-0.2394	0.2121	0.0713	3.193	31.68%
	(0.072)	(0.672)	(0.512)	(0.923)	(0.32)	(0.756)	**(0.095)**	(0.444)	**(0.043)**	(2.90%)
3 Log stock excess return	2.1727	-0.8915	-0.2858	0.0239	0.2973	-0.9771	0.2125	0.4429	1.7771	40.48%
	(0.003)	(0.557)	(0.355)	(0.890)	(0.37)	(0.375)	(0.236)	**(0.002)**	(0.420)	(0.30%)
4 Log bond excess return	0.6427	0.3368	-0.0495	-0.0372	-0.1681	0.2247	0.2203	0.0153	1.4751	33.47%
	(0.071)	(0.662)	(0.751)	(0.671)	(0.318)	(0.687)	**(0.018)**	(0.820)	(0.190)	(1.90%)
5 Log inflation rate	-0.0748	0.171	0.0511	-0.0487	-0.0635	0.6446	-0.0383	0.0088	0.4586	80.14%
	(0.332)	(0.314)	(0.141)	**(0.015)**	**(0.09)**	**(0.000)**	**(0.059)**	(0.550)	**(0.066)**	(0.00%)
6 Log of cap rate	-1.2729	0.1767	-0.24	-0.0214	-0.1471	0.5446	0.667	-0.0968	-3.7854	54.69%
	(0.029)	(0.887)	(0.343)	(0.880)	(0.587)	(0.546)	**(0.000)**	(0.374)	**(0.040)**	(0.00%)
7 Log of dividend yield	-2.1533	0.8172	0.2958	0.0467	-0.3888	0.6303	-0.3124	0.6364	-0.5733	61.81%
	(0.004)	(0.603)	(0.354)	(0.794)	(0.258)	(0.579)	**(0.095)**	**(0.000)**	(0.801)	(0.00%)
8 Log yield spread	0.0161	-0.0876	-0.0128	-0.0246	0.0224	0.0978	0.0055	-0.0015	0.3436	33.88%
	(0.744)	(0.421)	(0.561)	**(0.051)**	(0.344)	(0.217)	(0.664)	(0.878)	**(0.033)**	(1.70%)

Panel B: Standard deviations and correlations of VAR residuals

	1	2	3	4	5	6	7	8
1 Log nominal cash return	1.24%	-17.86%	-15.61%	-30.15%	60.31%	14.99%	18.88%	-43.29%
2 Log real estate excess return	-17.86%	14.15%	58.69%	19.75%	-3.06%	-96.82%	-56.47%	-25.59%
3 Log stock excess return	-15.61%	58.69%	20.16%	41.90%	-15.93%	-55.83%	-95.54%	-23.89%
4 Log bond excess return	-30.15%	19.75%	41.90%	10.23%	-55.24%	-13.85%	-45.94%	-5.59%
5 Log inflation	60.31%	-3.06%	-15.93%	-55.24%	2.25%	0.05%	19.73%	-12.23%
6 Log of cap rate	14.99%	-96.82%	-55.83%	-13.85%	0.05%	16.54%	54.47%	23.00%
7 Log of dividend yield	18.88%	-56.47%	-95.54%	-45.94%	19.73%	54.47%	20.85%	25.13%
8 Log yield spread	-43.29%	-25.59%	-23.89%	-5.59%	-12.23%	23.00%	25.13%	1.44%

Table 3: Results obtained from the use of alternative smoothing parameters

This table shows results for three parameters *a* used to unsmooth real estate returns and four investment horizons. Results are obtained from re-estimated VARs where the real estate excess return and cap rate series are based on the alternative assumptions.

Investment horizon (years)	1			5			10			20		
Smoothing parameter *a*	0.5	0.625	0.75	0.5	0.625	0.75	0.5	0.625	0.75	0.5	0.625	0.75
Expected return on real estate p.a.												
Real return	-1.23%	-1.91%	-2.07%	3.20%	2.93%	3.02%	3.50%	3.30%	3.42%	3.61%	3.43%	3.56%
Nominal return	4.38%	3.70%	3.55%	9.01%	8.74%	8.83%	9.58%	9.31%	9.51%	9.99%	9.82%	9.96%
Conditional standard deviation of real estate returns p.a.												
Real return	17.60%	14.11%	11.77%	11.41%	10.09%	9.33%	9.20%	8.56%	7.96%	8.11%	7.71%	7.52%
Nominal return	17.43%	13.98%	11.67%	11.54%	10.46%	9.94%	10.60%	10.08%	9.90%	11.23%	10.95%	10.94%
Conditional correlation of inflation and real estate returns												
Nominal return	-0.97%	2.27%	4.95%	30.57%	37.36%	42.65%	59.94%	64.87%	68.09%	77.43%	79.53%	80.98%
Real estate weight at global minimum variance portfolio												
Real return	1.51%	2.19%	3.02%	14.74%	16.30%	17.04%	27.56%	31.39%	33.41%	38.08%	42.48%	45.05%
Nominal return	1.12%	1.36%	1.65%	5.35%	6.26%	6.50%	6.71%	9.54%	11.41%	18.30%	25.53%	32.53%
Real estate weight at portfolio with $\gamma = 5$												
Real return	0.00%	0.00%	0.00%	31.75%	36.69%	42.92%	45.06%	48.25%	54.16%	51.17%	50.59%	55.72%
Nominal return	0.00%	0.00%	0.00%	14.94%	18.01%	23.07%	14.42%	15.72%	20.85%	25.10%	25.65%	32.35%

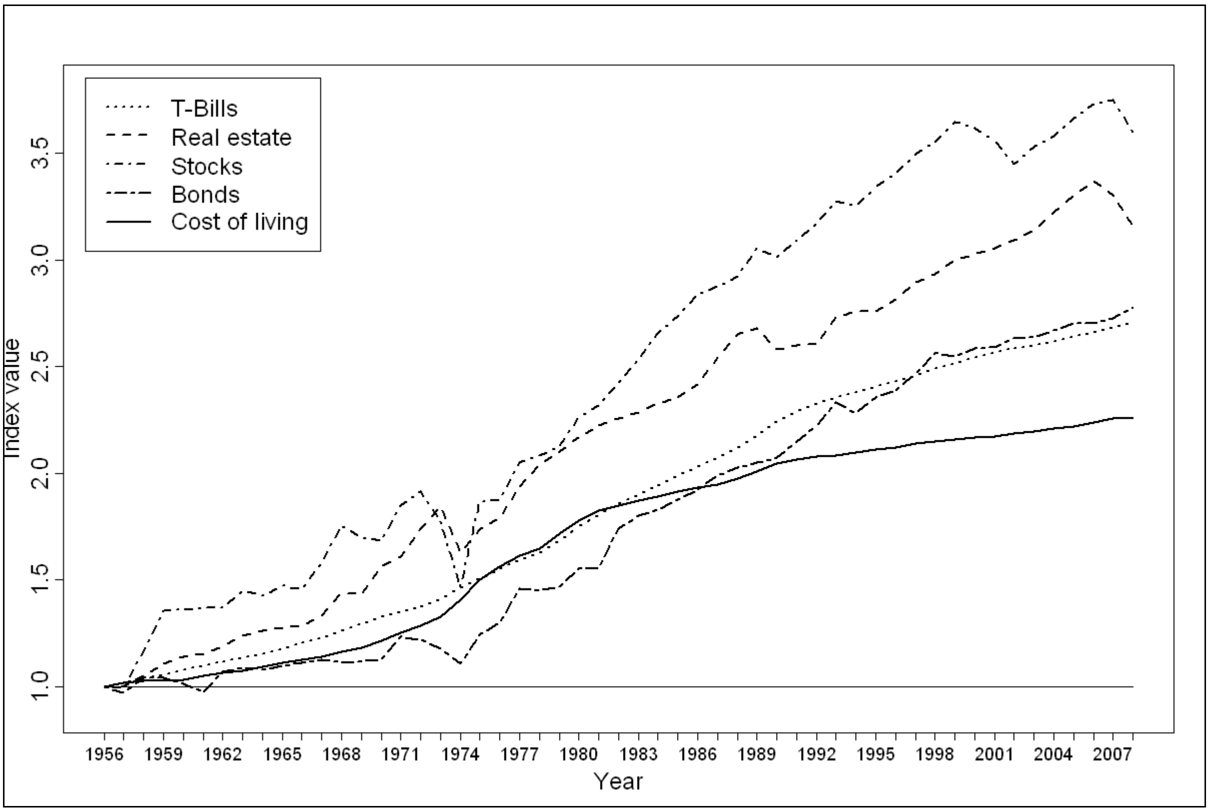


Figure 1: Total return and cost of living indexes
The figure shows the logarithm of the nominal total return index values and the cost of living index over the time period 1956 to 2008 (end of 1956 = 1).

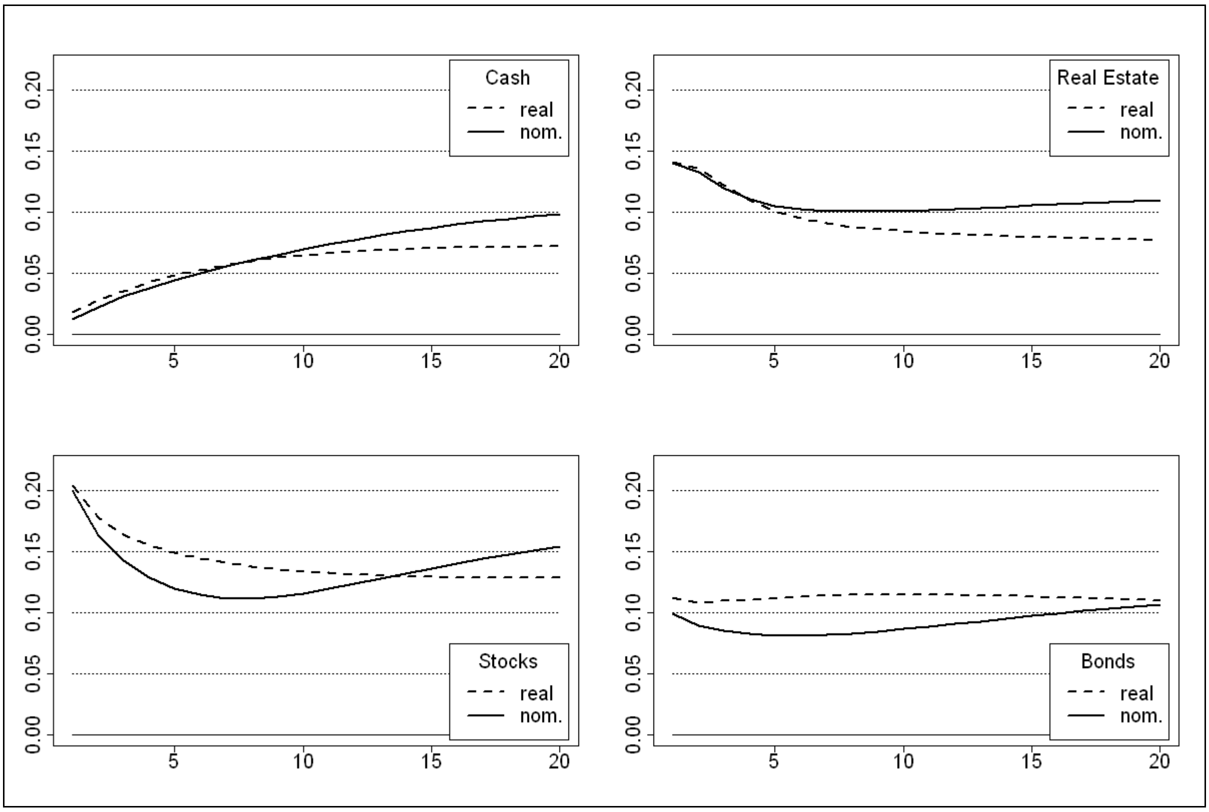


Figure 2: The term structure of return volatilities
The figure shows the annualized conditional standard deviations of real and nominal returns of the four assets depending on the investment horizon (years).

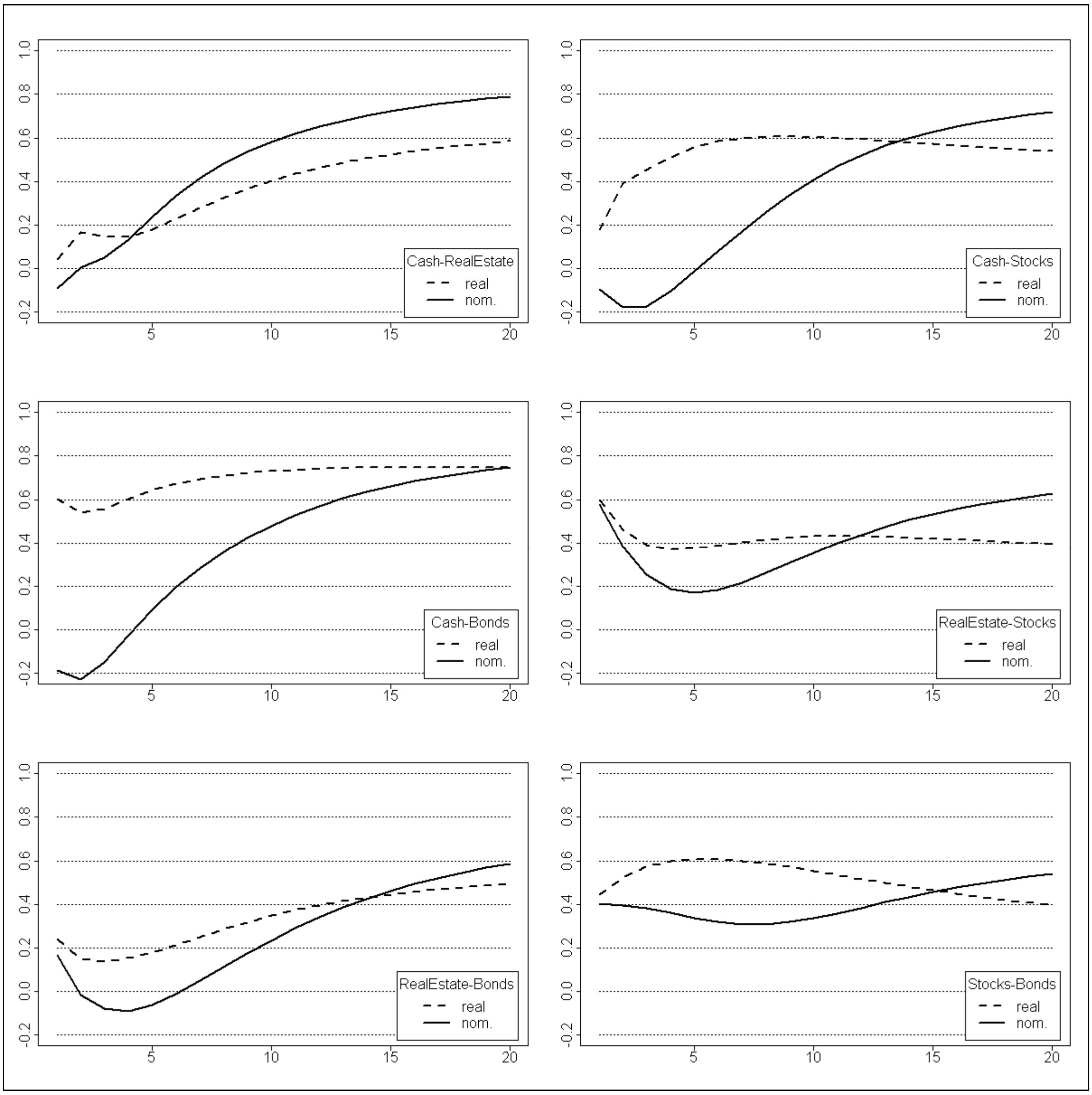


Figure 3: The term structure of return correlations
The figure shows conditional return correlations depending on the investment horizon (years) for both nominal and real returns.

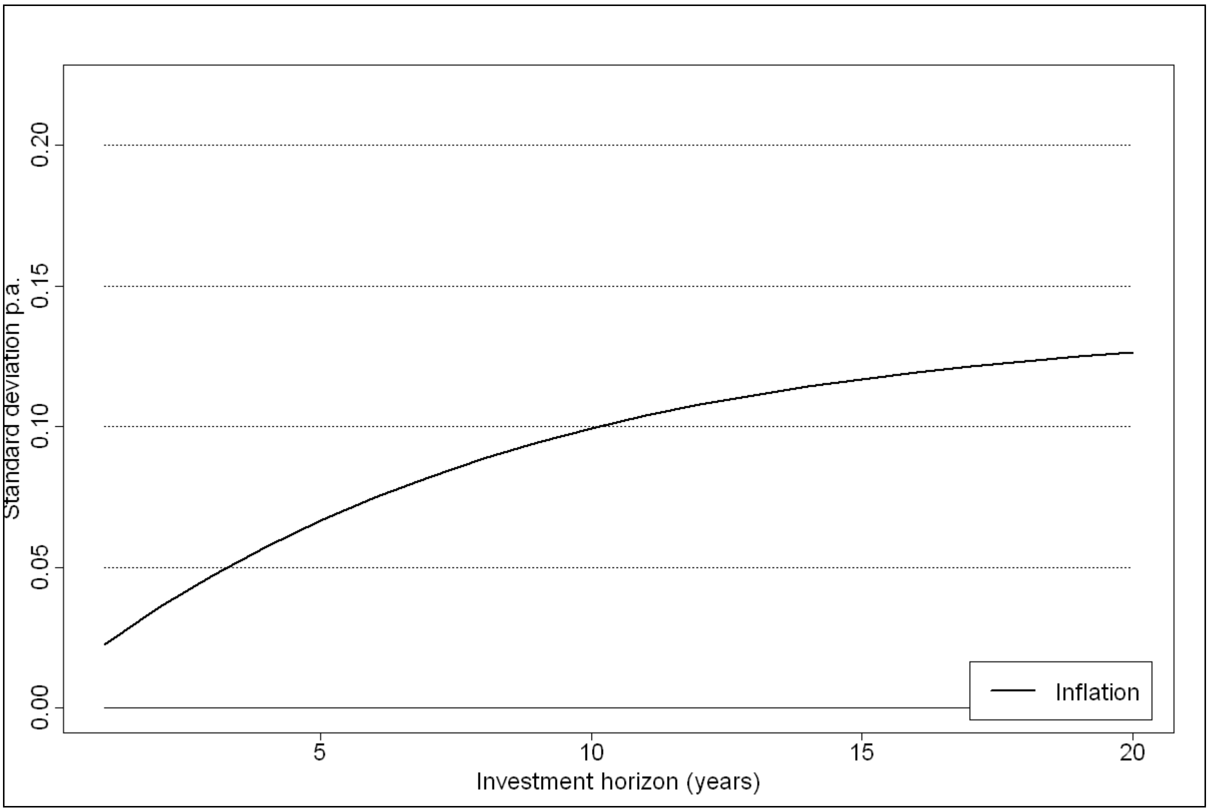


Figure 4: The term structure of inflation volatility
The figure shows the conditional annualized standard deviation of inflation depending on the investment horizon.

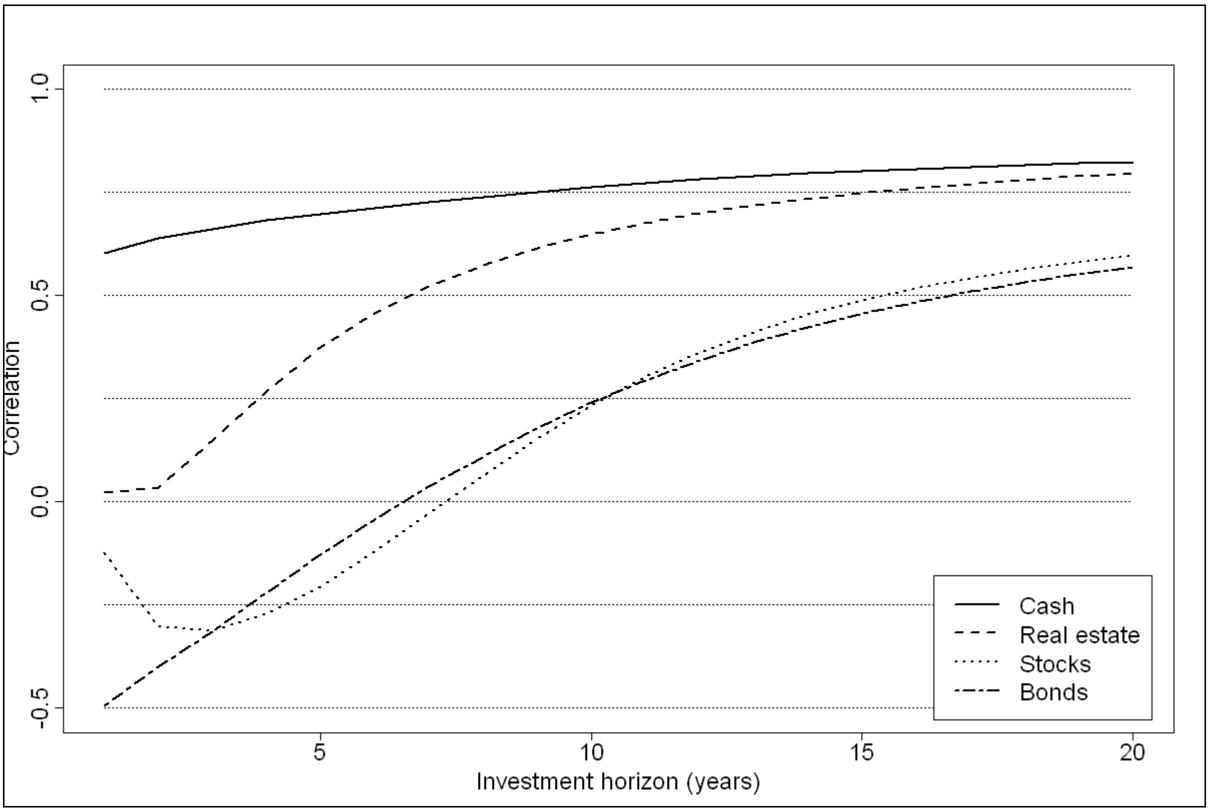


Figure 5: Inflation hedge properties
The figure shows conditional correlations of nominal returns and inflation depending on the investment horizon.

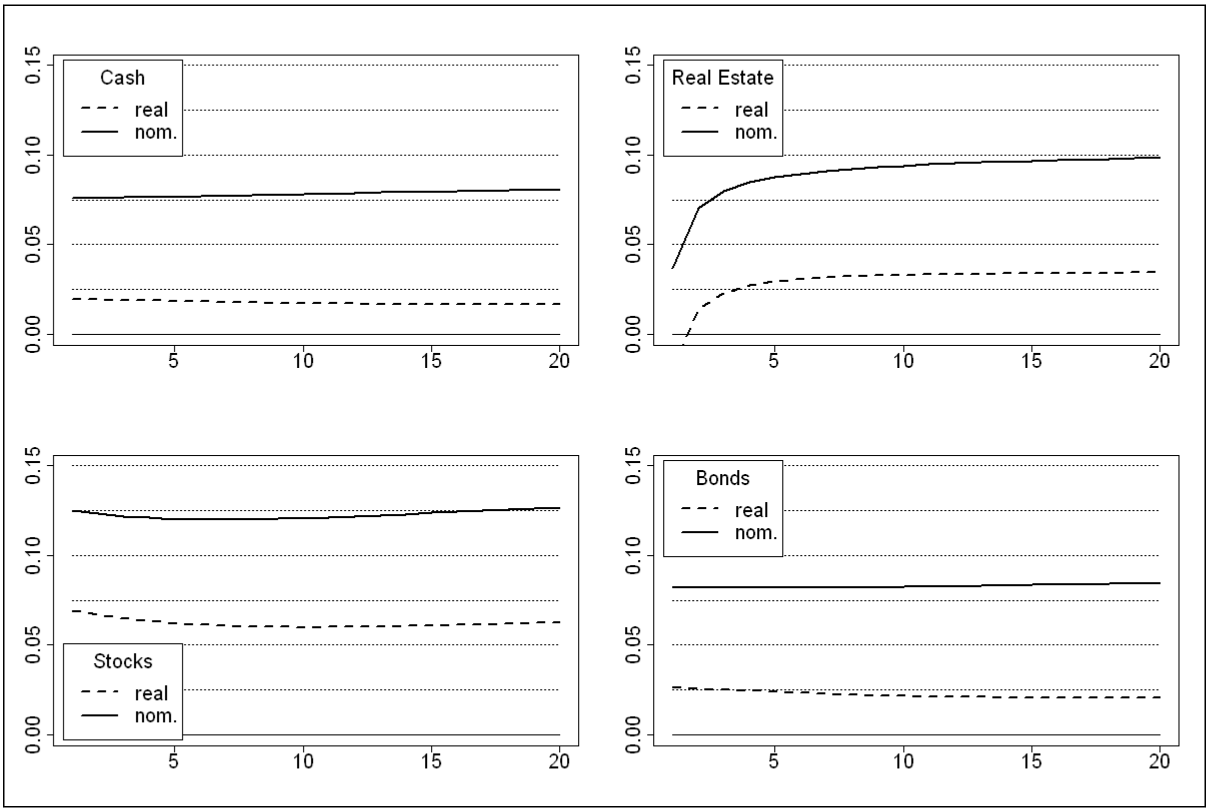
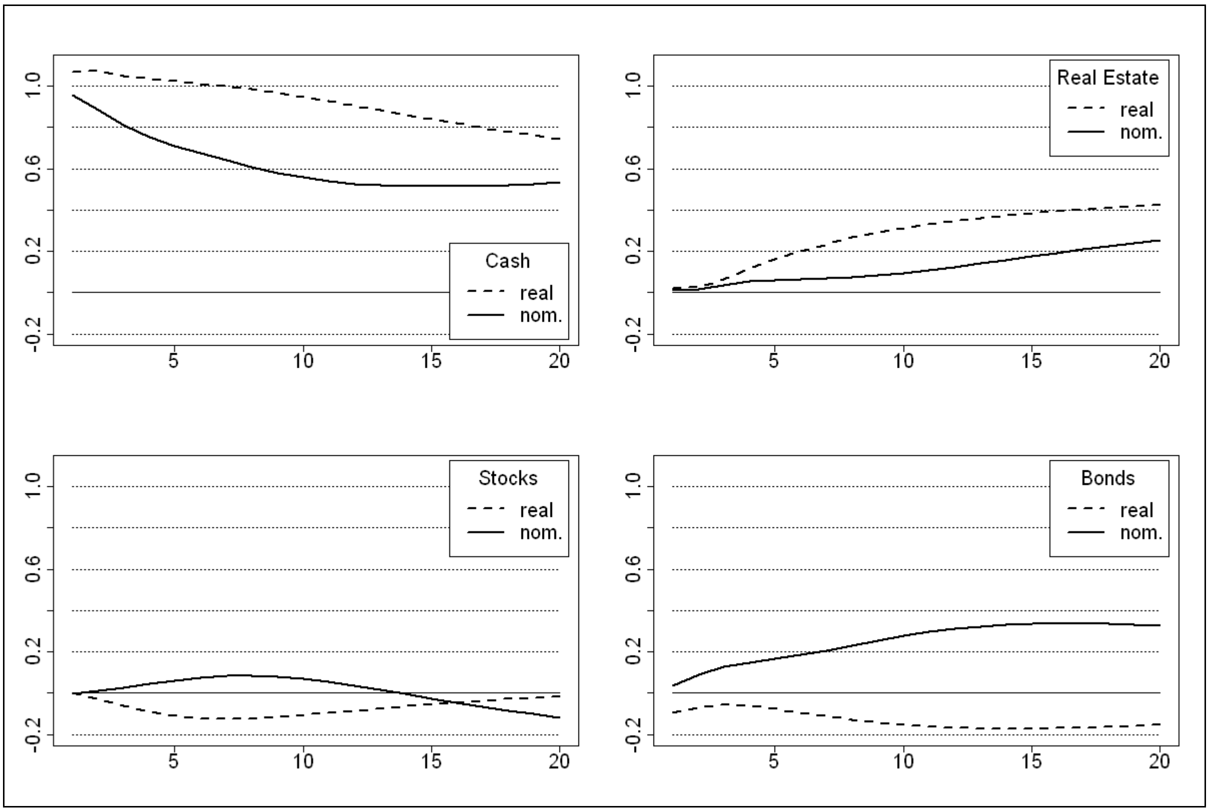
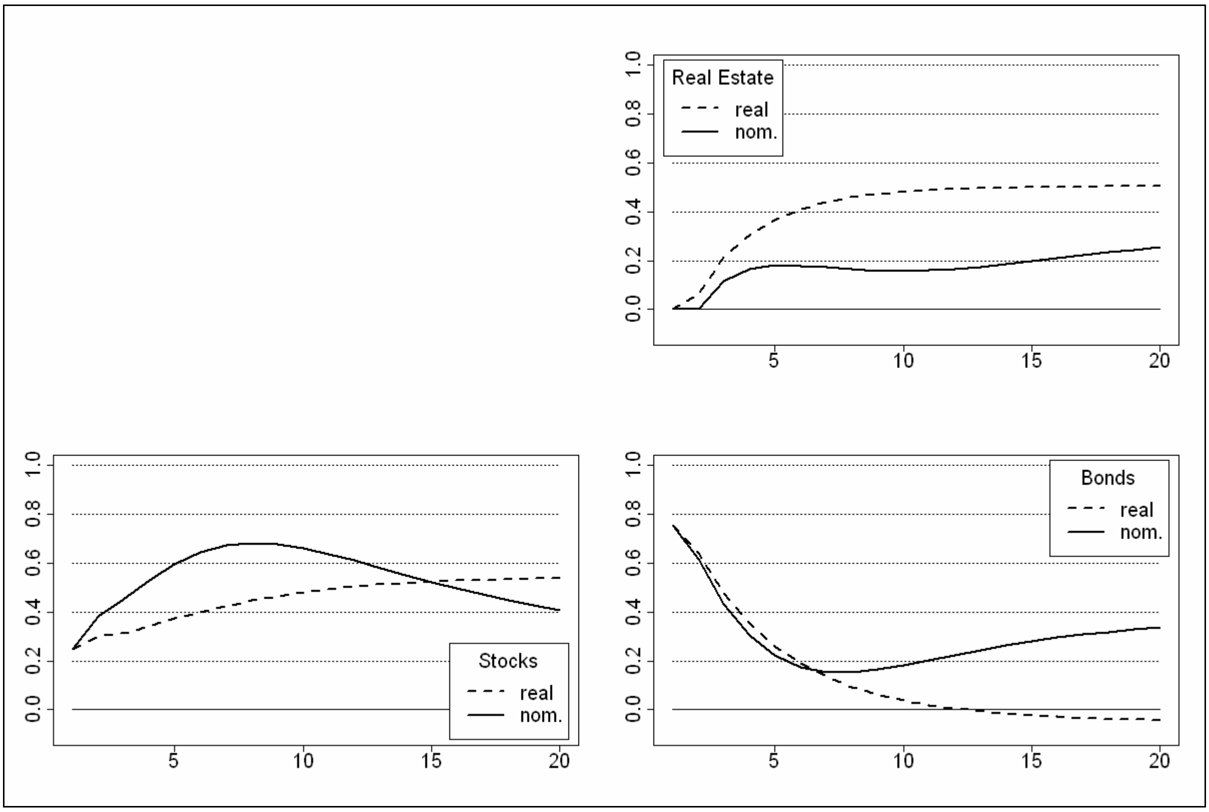


Figure 6: The term structure of expected returns

The figure shows annualized expected real and nominal returns depending on the investment horizon (years). These follow from (13a) and (13b), assuming a 100% investment in the respective asset. Expected log returns are assumed to equal their sample counterparts. Round-trip transaction costs are assumed to be 6.84% for real estate, 1.0% for stocks and 0.1% for bonds.



Panel A: Global minimum variance portfolio



Panel B: Optimal portfolio allocation with real estate, stocks and bonds for $\gamma = 5$.

Figure 7: Optimal portfolio compositions
The figure shows optimal portfolio compositions for real and nominal returns depending on the investment horizon (years).